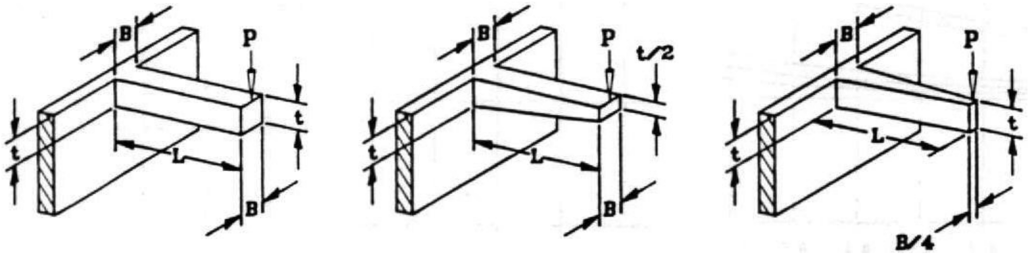
Calculations for Cantilever Beam Snap Fits



a) uniform section beam b) tapered height beam c) tapered width beam

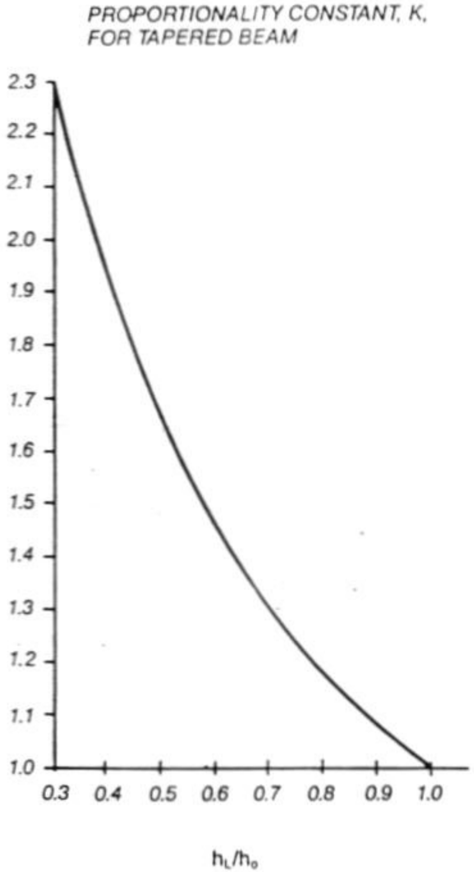
$$\varepsilon = \frac{3ty}{2L^2} \qquad \varepsilon = 0.92\frac{ty}{L^2} \qquad \varepsilon = 1.17\frac{ty}{L^2}$$

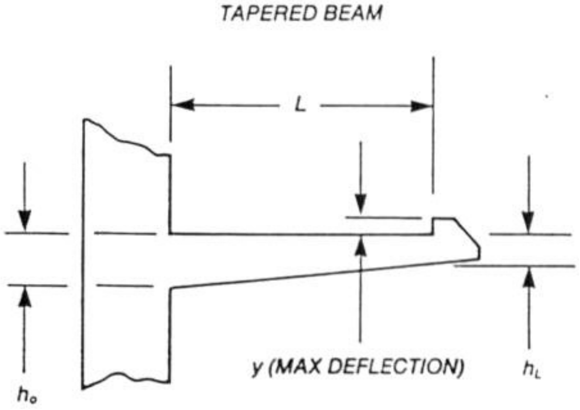
Figure 1: Dynamic strain calculations for various beam geometries (y = deflection).

When a snap fit is engaged, it stretches a certain amount, and then elastically recovers and returns to its original position. It is important to calculate the amount of strain that the plastic experiences. The calculated strain can then be compared to the material's properties so that a robust snap can be designed.

Figure 1 shows dynamic strain calculations for three different beam geometries. Figure 1a has a uniform rectangular cross section, 1b has a tapered height with a ratio of 2:1, and 1c has a tapered width with a ratio of 4:1. The formulas are shown below each figure. As can be seen from the formulas, Figure 1b is the design that results in the lowest amount of dynamic strain **(this is the recommended design for a cantilever beam snap fit)**. The design in Figure 1b results in a more uniform stress distribution, so it is able to bend more than the others at a lower stress and at a lower dynamic strain. The beam is tapered so that the base height is twice as big as the end height.

Calculating strain for tapered beams





Dynamic Strain

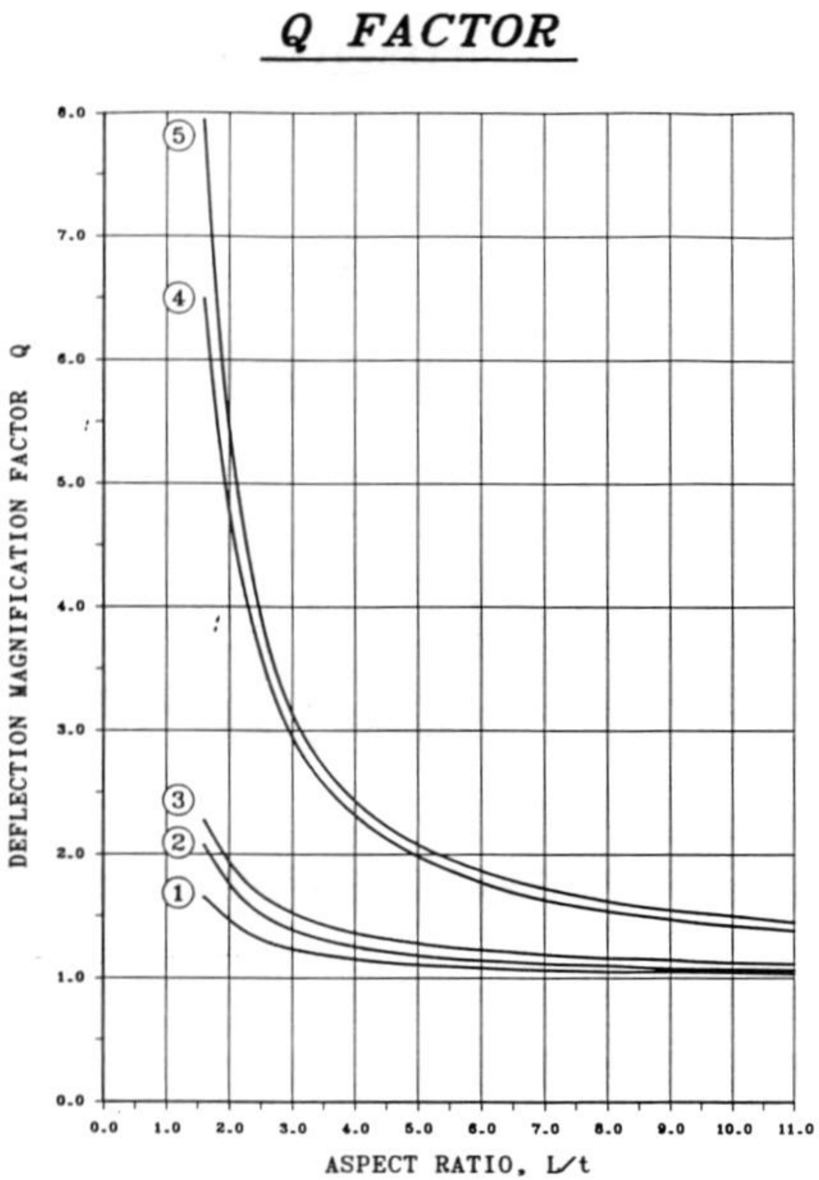
$$\varepsilon = \frac{3yh_0}{2L^2K}$$

Figure 2: Calculating dynamic strain for a tapered beam requires a proportionality constant.

When calculating the dynamic strain for a tapered beam, a proportionality constant needs to be added to the equation. The chart and picture in Figure 2 show how to find the constant of proportionality, K. First, find the ratio of the lengths by dividing h_L/h_0. Find this value on the x axis and find the corresponding K on the y axis. The new formula is shown in Figure 2; K is multiplied into the original equation's denominator.

As an example, refer to Figure 1b. $h_L = t/2$, and $h_0 = t$. $h_L/h_0 = 0.5$. From the chart, 0.5 on the x axis corresponds to a K of about 1.63. Substitute these values into the strain equation from Figure 2. $\varepsilon = \frac{3yt}{2L^2 * 1.63} = 0.92\frac{yt}{L^2}$. This the exact the formula shown in Figure 1b.

The Q Factor



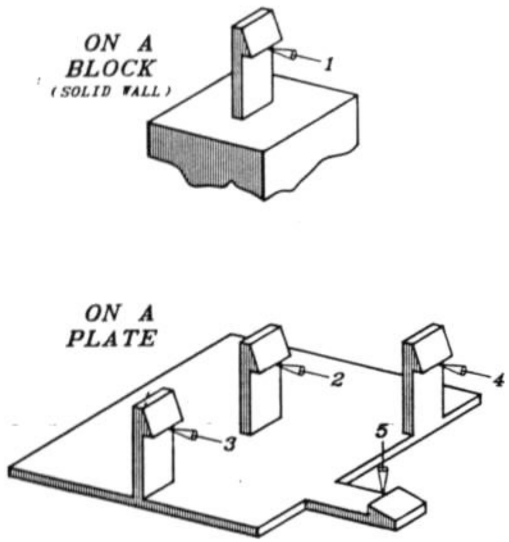


Figure 4: These scenarios correspond to the 5 lines in the Q factor graph in Figure 2.

Dynamic Strain with Q Factor

$$\varepsilon = \frac{3yt}{2L^2KQ}$$

Figure 3: The Q factor accounts for deflection of the wall to which the cantilever beam is attached. The circled

numbers refer to the scenarios shown in Figure 3.

The above formulas assume that the wall to which the beam is attached is completely stationary. This is not always a safe assumption, because the wall can deflect as the beam is pushed, significantly changing the amount of strain. To account for this, the Q factor is added to the equation. Refer to Figure 3 and Figure 4. Figure 3 shows the Q factor plots for the five different scenarios shown in Figure 4. To use the chart, calculate the aspect ratio of the beam (L/t, where t is the height of the beam's base). Next, refer to Figure 4 and find the appropriate beam. Go to the Q Factor chart, find the calculated aspect ratio on the x axis, and find the corresponding Q factor on the y axis.

Calculating Mating Force

The amount of force needed to put the snap fit together can be calculated. The amount of force to deflect the cantilever is given by the following equation: $P = \frac{2EI\varepsilon}{Lt}$, where ε is the strain that snap is designed to, and I is the moment of inertia. For a rectangular channel, $I = \frac{Bt^3}{12}$, where B is the depth of the beam and t is the thickness.

The assembly force of the snap fit is a function of the engagement angle on the snap's hook, α, and the plastic's coefficient of friction, μ. Assembly force is given by the following equation: $W = P\frac{\mu + \tan\alpha}{1 - \mu\tan\alpha}$.

Safety Factors

The permissible strain for a snap fit design varies depending on the material properties. For materials with a distinct yield point, 0.7 times the elongation at yield can be used. For materials without a distinct yield point (usually fiber reinforced plastics), 0.5 times the elongation at break can be used. Some typical allowable strain values are listed in the table below.

Distinct yield point: $0.5\varepsilon_{break}$ = Design Strain

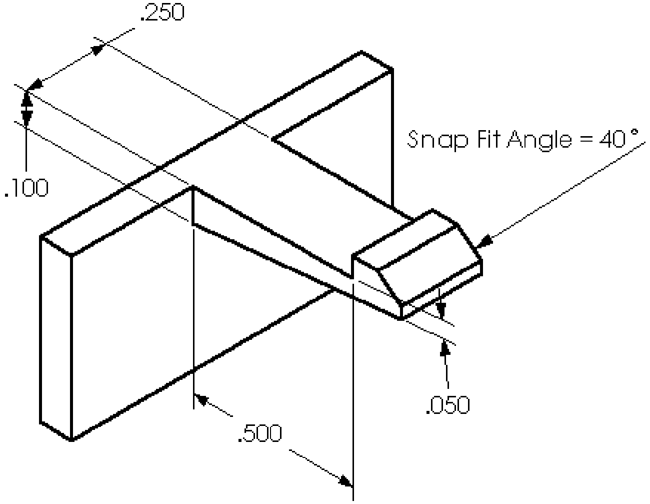
No distinct yield point: $0.7\varepsilon_{yield}$ = Design Strain

Typical Material Properties for Snap Fit Calculations

Material	Unreinforced Allowable Strain	30% Reinforced Allowable Strain	Coefficient of Friction (μ)
PEI	9.8%		0.20 - 0.25

PC	4% - 9.2%		0.25 - 0.30
Acetal	1.5%		0.20 - 0.35
Nylon 6	8%	2.1%	0.17 - 0.26
PBT	8.8%		0.35 - 0.40
PC/PET	5.8%		0.40 - 0.50
ABS	6% - 7%		0.50 - 0.60
PET		1.5%	0.18 - 0.25

Example #1



Material: Dupont Zytel 70G33HSIL, 33% Glass Filled Nylon 66
E = 1,520,000 psi (from Dupont's data sheet)
ε_{break} = 3% (from Dupont's data sheet)
ε_{design} = 1.5% (half of strain at break as a safety factor)
μ = 0.26 (from the table)

Find the maximum deflection of the snap. Find the mating force when assembling the snap, assuming that the beam is not tapered and has a constant thickness of 0.075".

We will use dynamic strain equation $\varepsilon = \dfrac{3yt}{2L^2KQ}$.

We need to solve for y to find the maximum deflection.

$$y = \frac{2L^2KQ\varepsilon}{3t}$$

Next, we need to find Q. The aspect ratio, L / t = 0.5 / 0.1 = 5. From Figure 4, our snap is like #4. From Figure 4, the correct Q = 2.

Next, we need to find K. Refer to Figure 2. Since this beam is tapered, we need to find the ratio of the end of the beam's thickness to the base of the beam's thickness. 0.050 / 0.100 = 0.5. K = 1.63.

Next, we plug the numbers into the formula and solve for y.

$$y = \frac{2(0.5)^2(1.63)(2)(0.015)}{3(0.1)}$$

y = 0.082"

The force to deflect the snap is given by the equation, $P = \dfrac{2EI\varepsilon}{LtQ}$. We need to find the moment of inertia (since a tapered beam would have a constantly changing moment of inertia, hand calculations would be extremely difficult, so we are assuming the beam has a constant thickness of 0.075"). The assumption of constant beam thickness may not be accurate; a finite element analysis would be needed for accurate results.

$I = Bt^3/12$

$I = (0.25)(0.10)^3/12$

$I = 2.083 \text{ x } 10^{-5}$

$$P = \frac{2(1{,}520{,}000)(2.083 \text{ x } 10^{-5})(0.015)}{(0.5)(0.10)(2)}$$

P = 9.5 lbf

The assembly force is given by the equation, $W = P\dfrac{\mu + \tan\alpha}{1 - \mu\tan\alpha}$.

$$W = 9.5\frac{0.26 + \tan 40^\circ}{1 - 0.26\tan 40^\circ}$$

W = 13.4 lbf

The amount of force could be reduced significantly by reducing the angle and/or decreasing the deflection of the beam.