8

Methods of Analysis and Selected Topics (dc)

8.1 INTRODUCTION

The circuits described in the previous chapters had only one source or two or more sources in series or parallel present. The step-by-step procedure outlined in those chapters cannot be applied if the sources are not in series or parallel. There will be an interaction of sources that will not permit the reduction technique used in Chapter 7 to find quantities such as the total resistance and source current.

Methods of analysis have been developed that allow us to approach, in a systematic manner, a network with any number of sources in any arrangement. Fortunately, these methods can also be applied to networks with only one source. The methods to be discussed in detail in this chapter include **branch-current analysis, mesh analysis,** and **nodal analysis.** Each can be applied to the same network. The "best" method cannot be defined by a set of rules but can be determined only by acquiring a firm understanding of the relative advantages of each. All the methods can be applied to *linear bilateral* networks. The term *linear* indicates that the characteristics of the network elements (such as the resistors) are independent of the voltage across or current through them. The second term, *bilateral,* refers to the fact that there is no change in the behavior or characteristics of an element if the current through or voltage across the element is reversed. Of the three methods listed above, the branch-current method is the only one not restricted to bilateral devices. Before discussing the methods in detail, we shall consider the current source and conversions between voltage and current sources. At the end of the chapter we shall consider bridge networks and Δ-Y and Y-Δ conversions. Chapter 9 will present the important theorems of network analysis that can also be employed to solve networks with more than one source.

8.2 CURRENT SOURCES

The concept of the **current source** was introduced in Section 2.4 with the photograph of a commercially available unit. We must now investigate its characteristics in greater detail so that we can properly determine its effect on the networks to be examined in this chapter.

The current source is often referred to as the *dual* of the voltage source. A battery supplies a *fixed* voltage, and the source current can vary; but the current source supplies a *fixed* current to the branch in which it is located, while its terminal voltage may vary as determined by the network to which it is applied. Note from the above that *duality* simply implies an interchange of current and voltage to distinguish the characteristics of one source from the other.

The interest in the current source is due primarily to semiconductor devices such as the transistor. In the basic electronics courses, you will find that the transistor is a current-controlled device. In the physical model (equivalent circuit) of a transistor used in the analysis of transistor networks, there appears a current source as indicated in Fig. 8.1. The symbol for a current source appears in Fig. 8.1(a). The direction of the arrow within the circle indicates the direction in which current is being supplied.

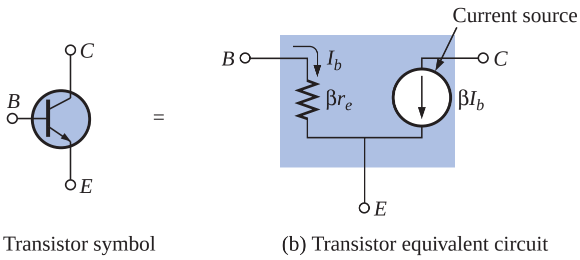


FIG. 8.1
Current source within the transistor equivalent circuit.

For further comparison, the terminal characteristics of an *ideal dc* voltage and current source are presented in Fig. 8.2, *ideal* implying perfect sources, or no internal losses sensitive to the demand from the applied load. Note that for the voltage source, the terminal voltage is fixed at E volts independent of the direction of the current I. The direction and magnitude of I will be determined by the network to which the supply is connected.

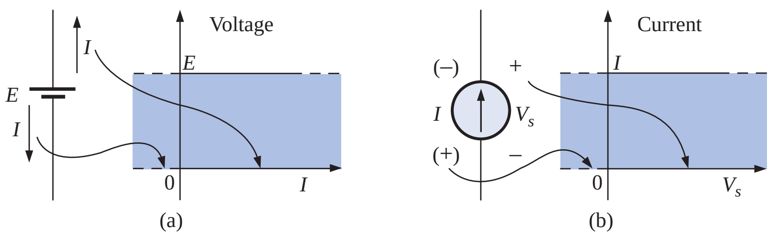


FIG. 8.2
Comparing the characteristics of an ideal voltage and current source.

The characteristics of the ideal current source, shown in Fig. 8.2(b), reveal that the magnitude of the supply current is independent of the polarity of the voltage across the source. The polarity and magnitude of the source voltage V_s will be determined by the network to which the source is connected.

For all one-voltage-source networks the current will have the direction indicated to the right of the battery in Fig. 8.2(a). For all single-current-source networks, it will have the polarity indicated to the right of the current source in Fig. 8.2(b).

In review:

A current source determines the current in the branch in which it is located

and

the magnitude and polarity of the voltage across a current source are a function of the network to which it is applied.

EXAMPLE 8.1 Find the source voltage V_s and the current I_1 for the circuit of Fig. 8.3.

Solution:

$$I_1 = I = \mathbf{10\ mA}$$
$$V_s = V_1 = I_1R_1 = (10\ \text{mA})(20\ \text{k}\Omega) = \mathbf{200\ V}$$

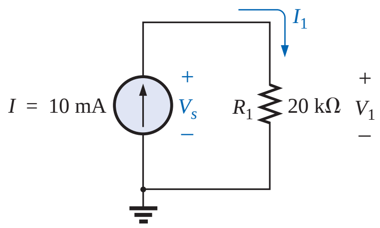


FIG. 8.3
Example 8.1.

EXAMPLE 8.2 Find the voltage V_s and the currents I_1 and I_2 for the network of Fig. 8.4.

Solution:

$$V_s = E = \mathbf{12\ V}$$
$$I_2 = \frac{V_R}{R} = \frac{E}{R} = \frac{12\ \text{V}}{4\ \Omega} = \mathbf{3\ A}$$

Applying Kirchhoff's current law:

$$I = I_1 + I_2$$

and $$I_1 = I - I_2 = 7\ \text{A} - 3\ \text{A} = \mathbf{4\ A}$$

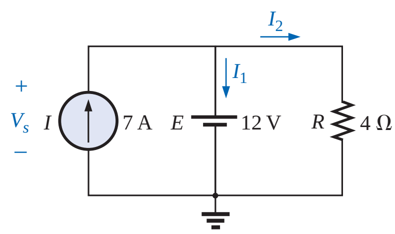


FIG. 8.4
Example 8.2.

EXAMPLE 8.3 Determine the current I_1 and the voltage V_s for the network of Fig. 8.5.

Solution: Using the current divider rule:

$$I_1 = \frac{R_2I}{R_2 + R_1} = \frac{(1\ \Omega)(6\ \text{A})}{1\ \Omega + 2\ \Omega} = \mathbf{2\ A}$$

The voltage V_1 is

$$V_1 = I_1R_1 = (2\ \text{A})(2\ \Omega) = 4\ \text{V}$$

and, applying Kirchhoff's voltage law,

$$+V_s - V_1 - 20\ \text{V} = 0$$

and $$V_s = V_1 + 20\ \text{V} = 4\ \text{V} + 20\ \text{V} = \mathbf{24\ V}$$

Note the polarity of V_s as determined by the multisource network.

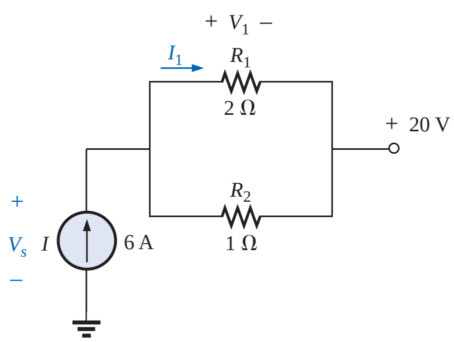


FIG. 8.5
Example 8.3.

8.3 SOURCE CONVERSIONS

The current source described in the previous section is called an *ideal source* due to the absence of any internal resistance. In reality, all

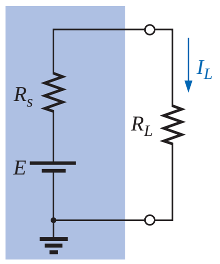


FIG. 8.6
Practical voltage source.

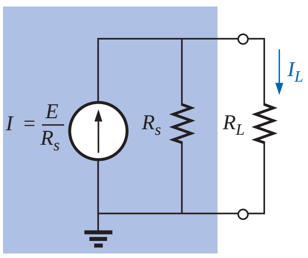


FIG. 8.7
Practical current source.

sources—whether they are voltage or current—have some internal resistance in the relative positions shown in Figs. 8.6 and 8.7. For the voltage source, if $R_s = 0\ \Omega$ or is so small compared to any series resistor that it can be ignored, then we have an "ideal" voltage source. For the current source, if $R_s = \infty\ \Omega$ or is large enough compared to other parallel elements that it can be ignored, then we have an "ideal" current source.

If the internal resistance is included with either source, then that source can be converted to the other type using the procedure to be described in this section. Since it is often advantageous to make such a maneuver, this entire section is devoted to being sure that the steps are understood. It is important to realize, however, as we proceed through this section, that

source conversions are equivalent only at their external terminals.

The internal characteristics of each are quite different.

We want the equivalence to ensure that the applied load of Figs. 8.6 and 8.7 will receive the same current, voltage, and power from each source and in effect not know, or care, which source is present.

In Fig. 8.6 if we solve for the load current I_L, we obtain

$$I_L = \frac{E}{R_s + R_L} \tag{8.1}$$

If we multiply this by a factor of 1, which we can choose to be R_s/R_s, we obtain

$$I_L = \frac{(1)E}{R_s + R_L} = \frac{(R_s/R_s)E}{R_s + R_L} = \frac{R_s(E/R_s)}{R_s + R_L} = \frac{R_s I}{R_s + R_L} \tag{8.2}$$

If we define $I = E/R_s$, Equation (8.2) is the same as that obtained by applying the current divider rule to the network of Fig. 8.7. The result is an equivalence between the networks of Figs. 8.6 and 8.7 that simply requires that $I = E/R_s$ and the series resistor R_s of Fig. 8.6 be placed in parallel, as in Fig. 8.7. The validity of this is demonstrated in Example 8.4 of this section.

For clarity, the equivalent sources, *as far as terminals a and b are concerned,* are repeated in Fig. 8.8 with the equations for converting in either direction. Note, as just indicated, that the resistor R_s is the same in each source; only its position changes. The current of the current source or the voltage of the voltage source is determined using Ohm's law and the parameters of the other configuration. It was pointed out in some detail in Chapter 6 that every source of voltage has some internal series resistance. *For the current source, some internal parallel resistance will always exist in the practical world.* However, in many cases, it is an

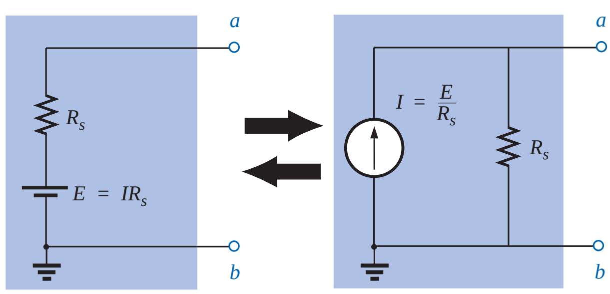


FIG. 8.8
Source conversion.

excellent approximation to drop the internal resistance of a source due to the magnitude of the elements of the network to which it is applied. For this reason, in the analyses to follow, voltage sources may appear without a series resistor, and current sources may appear without a parallel resistance. Realize, however, that for us to perform a conversion from one type of source to another, a voltage source must have a resistor in series with it, and a current source must have a resistor in parallel.

EXAMPLE 8.4

a. Convert the voltage source of Fig. 8.9(a) to a current source, and calculate the current through the 4-Ω load for each source.
b. Replace the 4-Ω load with a 1-kΩ load, and calculate the current I_L for the voltage source.
c. Repeat the calculation of part (b) assuming that the voltage source is ideal ($R_s = 0\ \Omega$) because R_L is so much larger than R_s. Is this one of those situations where assuming that the source is ideal is an appropriate approximation?

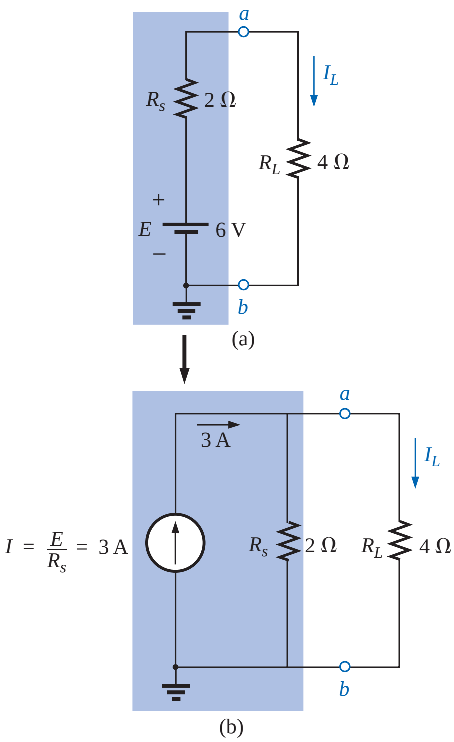


FIG. 8.9
Example 8.4.

Solutions:

a. See Fig. 8.9.

$$\text{Fig. 8.9(a):}\quad I_L = \frac{E}{R_s + R_L} = \frac{6\text{ V}}{2\ \Omega + 4\ \Omega} = \mathbf{1\ A}$$

$$\text{Fig. 8.9(b):}\quad I_L = \frac{R_s I}{R_s + R_L} = \frac{(2\ \Omega)(3\text{ A})}{2\ \Omega + 4\ \Omega} = \mathbf{1\ A}$$

b. $$I_L = \frac{E}{R_s + R_L} = \frac{6\text{ V}}{2\ \Omega + 1\text{ k}\Omega} \cong \mathbf{5.99\ mA}$$

c. $$I_L = \frac{E}{R_L} = \frac{6\text{ V}}{1\text{ k}\Omega} = \mathbf{6\ mA} \cong 5.99\text{ mA}$$

Yes, $R_L \gg R_s$ (voltage source).

EXAMPLE 8.5

a. Convert the current source of Fig. 8.10(a) to a voltage source, and find the load current for each source.
b. Replace the 6-kΩ load with a 10-Ω load, and calculate the current I_L for the current source.
c. Repeat the calculation of part (b) assuming that the current source is ideal ($R_s = \infty\ \Omega$) because R_L is so much smaller than R_s. Is this one of those situations where assuming that the source is ideal is an appropriate approximation?

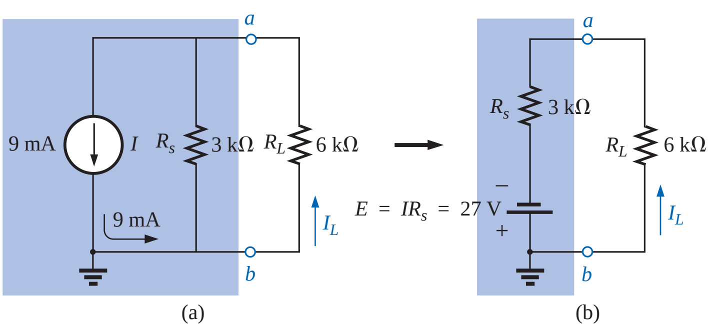


FIG. 8.10
Example 8.5.

Solutions:

a. See Fig. 8.10.

Fig. 8.10(a): $I_L = \dfrac{R_s I}{R_s + R_L} = \dfrac{(3\text{ k}\Omega)(9\text{ mA})}{3\text{ k}\Omega + 6\text{ k}\Omega} = \mathbf{3\text{ mA}}$

Fig. 8.10(b): $I_L = \dfrac{E}{R_s + R_L} = \dfrac{27\text{ V}}{3\text{ k}\Omega + 6\text{ k}\Omega} = \dfrac{27\text{ V}}{9\text{ k}\Omega} = \mathbf{3\text{ mA}}$

b. $I_L = \dfrac{R_s I}{R_s + R_L} = \dfrac{(3\text{ k}\Omega)(9\text{ mA})}{3\text{ k}\Omega + 10\ \Omega} = \mathbf{8.97\text{ mA}}$

c. $I_L = I = \mathbf{9\text{ mA}} \cong 8.97\text{ mA}$

Yes, $R_s \gg R_L$ (current source).

8.4 CURRENT SOURCES IN PARALLEL

If two or more current sources are in parallel, they may all be replaced by one current source having the magnitude and direction of the resultant, which can be found by summing the currents in one direction and subtracting the sum of the currents in the opposite direction. The new parallel resistance is determined by methods described in the discussion of parallel resistors in Chapter 5. Consider the following examples.

EXAMPLE 8.6 Reduce the parallel current sources of Figs. 8.11 and 8.12 to a single current source.

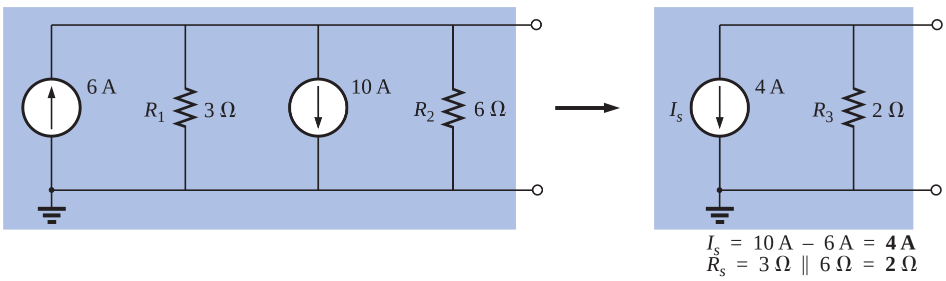


FIG. 8.11
Example 8.6.

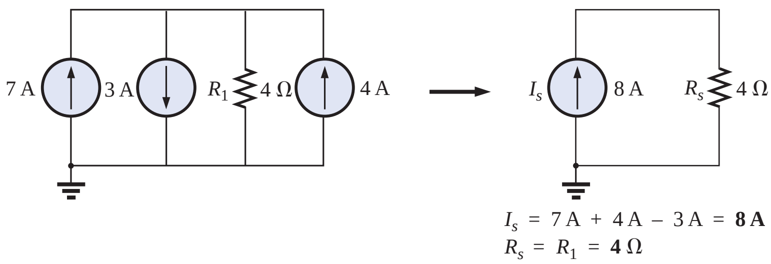


FIG. 8.12
Example 8.6.

Solution: Note the solution in each figure.

EXAMPLE 8.7 Reduce the network of Fig. 8.13 to a single current source, and calculate the current through R_L.

Solution: In this example, the voltage source will first be converted to a current source as shown in Fig. 8.14. Combining current sources,

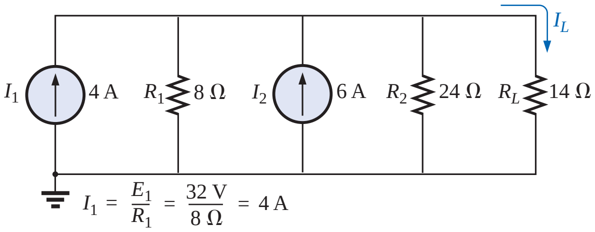


FIG. 8.14
Network of Fig. 8.13 following the conversion of the voltage source to a current source.

$$I_s = I_1 + I_2 = 4\text{ A} + 6\text{ A} = \mathbf{10\ A}$$

and $$R_s = R_1 \parallel R_2 = 8\ \Omega \parallel 24\ \Omega = \mathbf{6\ \Omega}$$

Applying the current divider rule to the resulting network of Fig. 8.15,

$$I_L = \frac{R_s I_s}{R_s + R_L} = \frac{(6\ \Omega)(10\text{ A})}{6\ \Omega + 14\ \Omega} = \frac{60\text{ A}}{20} = \mathbf{3\ A}$$

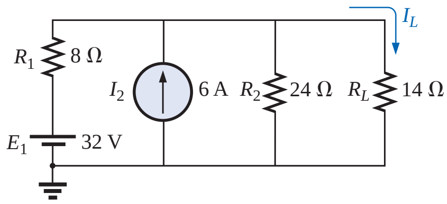


FIG. 8.13
Example 8.7.

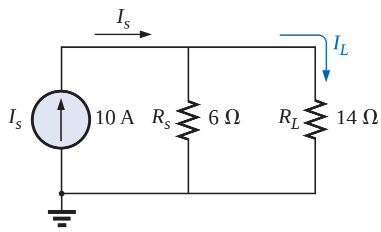


FIG. 8.15
Network of Fig. 8.14 reduced to its simplest form.

EXAMPLE 8.8 Determine the current I_2 in the network of Fig. 8.16.

Solution: Although it might appear that the network cannot be solved using methods introduced thus far, one source conversion as shown in Fig. 8.17 will result in a simple series circuit:

$$E_s = I_1 R_1 = (4\text{ A})(3\ \Omega) = 12\text{ V}$$

and $$R_s = R_1 = 3\ \Omega$$

and $$I_2 = \frac{E_s + E_2}{R_s + R_2} = \frac{12\text{ V} + 5\text{ V}}{3\ \Omega + 2\ \Omega} = \frac{17\text{ V}}{5\ \Omega} = \mathbf{3.4\ A}$$

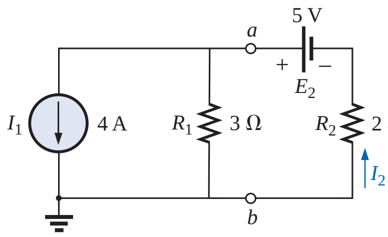


FIG. 8.16
Example 8.8.

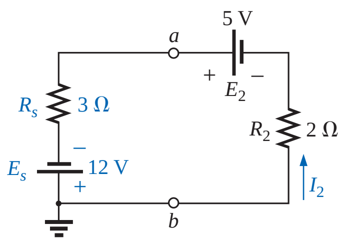


FIG. 8.17
Network of Fig. 8.16 following the conversion of the current source to a voltage source.

8.5 CURRENT SOURCES IN SERIES

The current through any branch of a network can be only single-valued. For the situation indicated at point *a* in Fig. 8.18, we find by application of Kirchhoff's current law that the current leaving that point is greater than that entering—an impossible situation. Therefore,

current sources of different current ratings are not connected in series,

just as voltage sources of different voltage ratings are not connected in parallel.

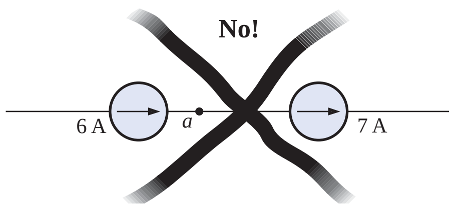


FIG. 8.18
Invalid situation.

8.6 BRANCH-CURRENT ANALYSIS

We will now consider the first in a series of methods for solving networks with two or more sources. Once the **branch-current method** is mastered, there is no linear dc network for which a solution cannot be found. Keep in mind that networks with two isolated voltage sources cannot be solved using the approach of Chapter 7. For additional evidence of this fact, try solving for the unknown elements of Example 8.9 using the methods introduced in Chapter 7. The network of Fig. 8.21 can be solved using the source conversions described in the last section, but the method to be described in this section has applications far beyond the configuration of this network. The most direct introduction to a method of this type is to list the series of steps required for its application. There are four steps, as indicated below. Before continuing, understand that this method will produce the current through each branch of the network, the *branch current.* Once this is known, all other quantities, such as voltage or power, can be determined.

1. ***Assign a distinct current of arbitrary direction to each branch of the network.***
2. ***Indicate the polarities for each resistor as determined by the assumed current direction.***
3. ***Apply Kirchhoff's voltage law around each closed, independent loop of the network.***

The best way to determine how many times Kirchhoff's voltage law will have to be applied is to determine the number of "windows" in the network. The network of Example 8.9 has a definite similarity to the two-window configuration of Fig. 8.19(a). The result is a need to apply Kirchhoff's voltage law twice. For networks with three windows, as shown in Fig. 8.19(b), three applications of Kirchhoff's voltage law are required, and so on.

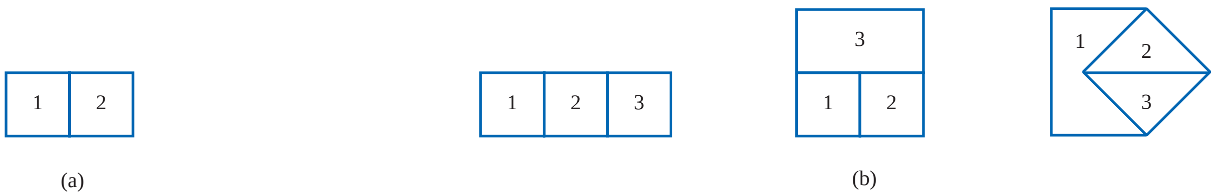


FIG. 8.19
Determining the number of independent closed loops.

4. ***Apply Kirchhoff's current law at the minimum number of nodes that will include all the branch currents of the network.***

The minimum number is one less than the number of independent nodes of the network. For the purposes of this analysis, a **node** is a junction of two or more branches, where a branch is any combination

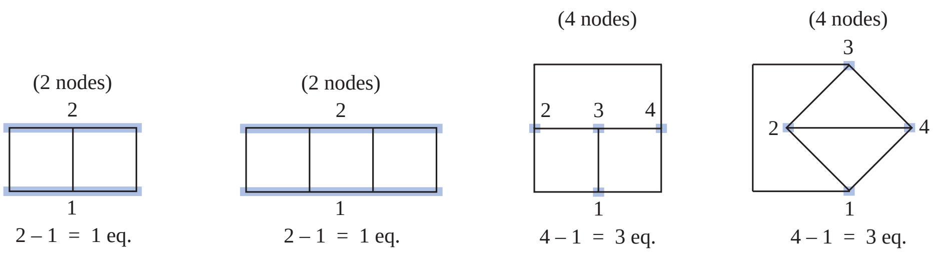


FIG. 8.20
Determining the number of applications of Kirchhoff's current law required.

of series elements. Figure 8.20 defines the number of applications of Kirchhoff's current law for each configuration of Fig. 8.19.

5. Solve the resulting simultaneous linear equations for assumed branch currents.

It is assumed that the use of the **determinants method** to solve for the currents I_1, I_2, and I_3 is understood and is a part of the student's mathematical background. If not, a detailed explanation of the procedure is provided in Appendix C. Calculators and computer software packages such as Mathcad can find the solutions quickly and accurately.

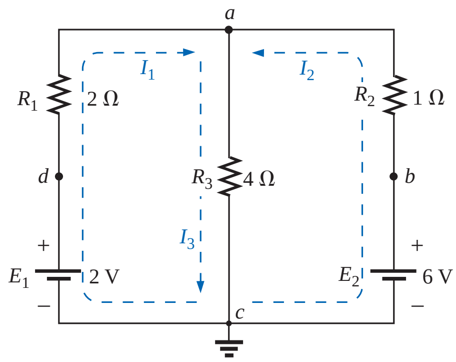


FIG. 8.21
Example 8.9.

EXAMPLE 8.9 Apply the branch-current method to the network of Fig. 8.21.

Solution 1:

Step 1: Since there are three distinct branches (*cda*, *cba*, *ca*), three currents of arbitrary directions (I_1, I_2, I_3) are chosen, as indicated in Fig. 8.21. The current directions for I_1 and I_2 were chosen to match the "pressure" applied by sources E_1 and E_2, respectively. Since both I_1 and I_2 enter node *a*, I_3 is leaving.

Step 2: Polarities for each resistor are drawn to agree with assumed current directions, as indicated in Fig. 8.22.

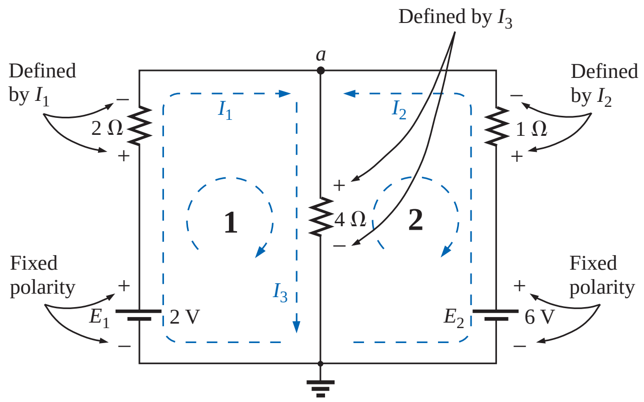


FIG. 8.22
Inserting the polarities across the resistive elements as defined by the chosen branch currents.

Step 3: Kirchhoff's voltage law is applied around each closed loop (1 and 2) in the clockwise direction:

Rise in potential

$$\text{loop 1:}\quad \Sigma_{\circlearrowright} V = +E_1 - V_{R_1} - V_{R_3} = 0$$

Drop in potential

Rise in potential

$$\text{loop 2:}\quad \Sigma_{\circlearrowright} V = +V_{R_3} + V_{R_2} - E_2 = 0$$

Drop in potential

and

$$\text{loop 1:}\quad \Sigma_{\circlearrowright} V = \underbrace{+2\text{ V}}_{\substack{\text{Battery}\\\text{potential}}} - \underbrace{(2\ \Omega)I_1}_{\substack{\text{Voltage drop}\\\text{across 2-}\Omega\\\text{resistor}}} - \underbrace{(4\ \Omega)I_3}_{\substack{\text{Voltage drop}\\\text{across 4-}\Omega\\\text{resistor}}} = 0$$

$$\text{loop 2:}\quad \Sigma_{\circlearrowright} V = (4\ \Omega)I_3 + (1\ \Omega)I_2 - 6\text{ V} = 0$$

Step 4: Applying Kirchhoff's current law at node *a* (in a two-node network, the law is applied at only one node),

$$I_1 + I_2 = I_3$$

Step 5: There are three equations and three unknowns (units removed for clarity):

$$\begin{aligned} 2 - 2I_1 - 4I_3 &= 0 \\ 4I_3 + 1I_2 - 6 &= 0 \\ I_1 + I_2 &= I_3 \end{aligned} \qquad \text{Rewritten:} \quad \begin{aligned} 2I_1 + 0 + 4I_3 &= 2 \\ 0 + I_2 + 4I_3 &= 6 \\ I_1 + I_2 - I_3 &= 0 \end{aligned}$$

Using third-order determinants (Appendix C), we have

$$I_1 = \frac{\begin{vmatrix} 2 & 0 & 4 \\ 6 & 1 & 4 \\ 0 & 1 & -1 \end{vmatrix}}{D} = -1\ \mathbf{A}$$

A negative sign in front of a branch current indicates only that the actual current is in the direction opposite to that assumed.

$$D = \begin{vmatrix} 2 & 0 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & -1 \end{vmatrix}$$

$$I_2 = \frac{\begin{vmatrix} 2 & 2 & 4 \\ 0 & 6 & 4 \\ 1 & 0 & -1 \end{vmatrix}}{D} = 2\ \mathbf{A}$$

$$I_3 = \frac{\begin{vmatrix} 2 & 0 & 2 \\ 0 & 1 & 6 \\ 1 & 1 & 0 \end{vmatrix}}{D} = 1\ \mathbf{A}$$

Mathcad Solution: Once you understand the procedure for entering the parameters, you can use Mathcad to solve determinants such as

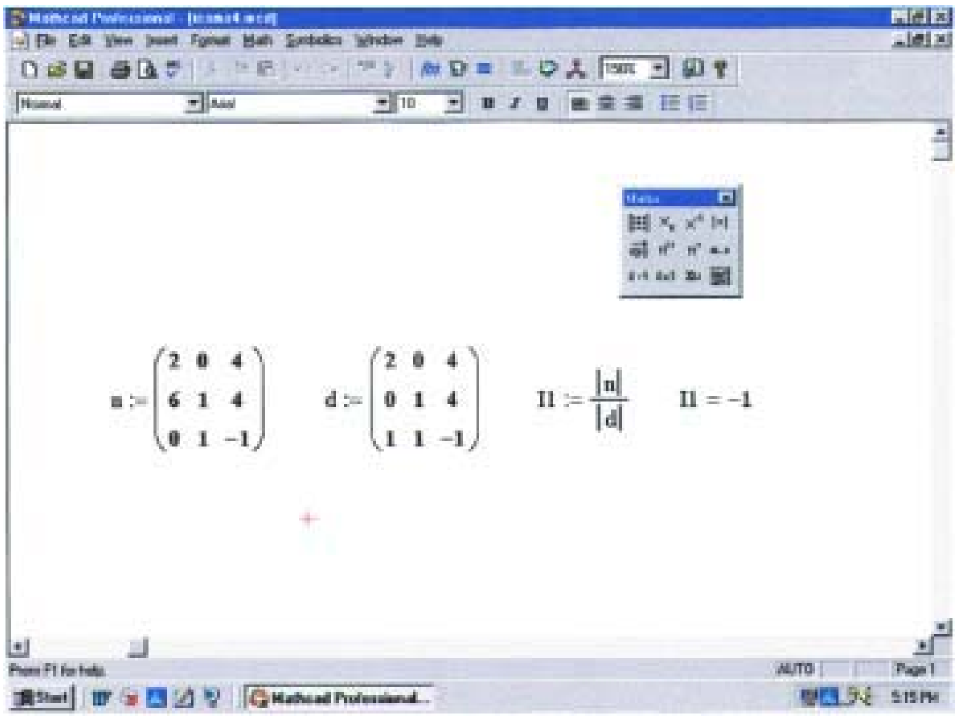


FIG. 8.23
Using Mathcad to verify the numerical calculations of Example 8.9.

appearing in Solution 1 in a very short time frame. The numerator is defined by *n* in the same manner described for earlier exercises. Then the sequence **View-Toolbars-Matrix** is applied to obtain the **Matrix** toolbar appearing in Fig. 8.23. Selecting the top left option called **Matrix** will result in the **Insert Matrix** dialog box in which **3 × 3** is selected. The 3 × 3 matrix will appear with a bracket to signal which parameter should be entered. Enter that parameter, and then use the left click of the mouse to select the next parameter you want to enter. When you have finished, move on to define the denominator *d* in the same manner. Then define the current of interest, select **Determinant** from the **Matrix** toolbar, and insert the numerator variable *n*. Follow with a division sign, and enter the **Determinant** of the denominator as shown in Fig. 8.23. Retype **I1** and select the equal sign; the correct result of **−1** will appear.

Once you have mastered the rather simple and direct process just described, the availability of Mathcad as a checking tool or solving mechanism will be deeply appreciated.

Solution 2: Instead of using third-order determinants as in Solution 1, we could reduce the three equations to two by substituting the third equation in the first and second equations:

$$\left.\begin{aligned} 2 - 2I_1 - 4\overbrace{(I_1 + I_2)}^{I_3} &= 0 \\ 4\overbrace{(I_1 + I_2)}^{I_3} + I_2 - 6 &= 0 \end{aligned}\right\} \quad \begin{aligned} 2 - 2I_1 - 4I_1 - 4I_2 &= 0 \\ 4I_1 + 4I_2 + I_2 - 6 &= 0 \end{aligned}$$

or

$$\begin{aligned} -6I_1 - 4I_2 &= -2 \\ +4I_1 + 5I_2 &= +6 \end{aligned}$$

Multiplying through by −1 in the top equation yields

$$\begin{aligned} 6I_1 + 4I_2 &= +2 \\ 4I_1 + 5I_2 &= +6 \end{aligned}$$

and using determinants,

$$I_1 = \frac{\begin{vmatrix} 2 & 4 \\ 6 & 5 \end{vmatrix}}{\begin{vmatrix} 6 & 4 \\ 4 & 5 \end{vmatrix}} = \frac{10 - 24}{30 - 16} = \frac{-14}{14} = \mathbf{-1\,A}$$

Using the TI-86 calculator:

det[[2,4][6,5]]/det[[6,4][4,5]] ENTER	−1

CALC. 8.1

Note the det (determinant) obtained from a Math listing under a MATRX menu and the fact that each determinant must be determined individually. The first set of brackets within the overall determinant brackets of the first determinant defines the first row of the determinant, while the second set of brackets within the same determinant defines the second row. A comma separates the entries for each row. Obviously, the time to learn how to enter the parameters is minimal when you consider the savings in time and the accuracy obtained.

$$I_2 = \frac{\begin{vmatrix} 6 & 2 \\ 4 & 6 \end{vmatrix}}{14} = \frac{36 - 8}{14} = \frac{28}{14} = \mathbf{2\ A}$$

$$I_3 = I_1 + I_2 = -1 + 2 = \mathbf{1\ A}$$

It is now important that the impact of the results obtained be understood. The currents I_1, I_2, and I_3 are the actual currents in the branches in which they were defined. A negative sign in the solution simply reveals that the actual current has the opposite direction than initially defined—the magnitude is correct. Once the actual current directions and their magnitudes are inserted in the original network, the various voltages and power levels can be determined. For this example, the actual current directions and their magnitudes have been entered on the original network in Fig. 8.24. Note that the current through the series elements R_1 and E_1 is 1 A; the current through R_3, 1 A; and the current through the series elements R_2 and E_2, 2 A. Due to the minus sign in the solution, the direction of I_1 is opposite to that shown in Fig. 8.21. The voltage across any resistor can now be found using Ohm's law, and the power delivered by either source or to any one of the three resistors can be found using the appropriate power equation.

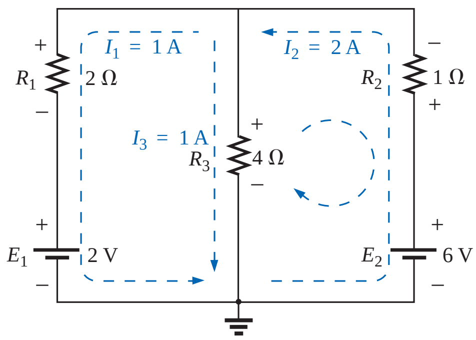


FIG. 8.24
Reviewing the results of the analysis of the network of Fig. 8.21.

Applying Kirchhoff's voltage law around the loop indicated in Fig. 8.24,

$$\Sigma_{\circlearrowright} V = +(4\ \Omega)I_3 + (1\ \Omega)I_2 - 6\ \text{V} = 0$$

or
$$(4\ \Omega)I_3 + (1\ \Omega)I_2 = 6\ \text{V}$$

and
$$\begin{aligned}(4\ \Omega)(1\ \text{A}) + (1\ \Omega)(2\ \text{A}) &= 6\ \text{V} \\ 4\ \text{V} + 2\ \text{V} &= 6\ \text{V} \\ 6\ \text{V} &= 6\ \text{V} \quad \text{(checks)}\end{aligned}$$

EXAMPLE 8.10 Apply branch-current analysis to the network of Fig. 8.25.

Solution: Again, the current directions were chosen to match the "pressure" of each battery. The polarities are then added and Kirchhoff's voltage law is applied around each closed loop in the clockwise direction. The result is as follows:

$$\text{loop 1:} \quad +15\ \text{V} - (4\ \Omega)I_1 + (10\ \Omega)I_3 - 20\ \text{V} = 0$$
$$\text{loop 2:} \quad +20\ \text{V} - (10\ \Omega)I_3 - (5\ \Omega)I_2 + 40\ \text{V} = 0$$

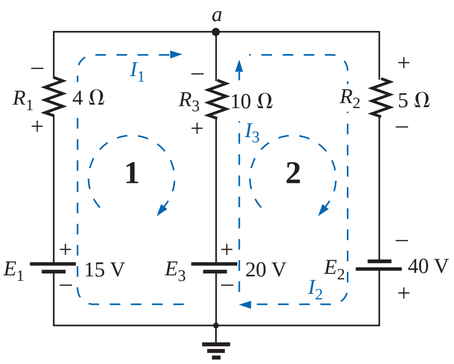


FIG. 8.25
Example 8.10.

Applying Kirchhoff's current law at node *a*,

$$I_1 + I_3 = I_2$$

Substituting the third equation into the other two yields (with units removed for clarity)

$$\left.\begin{aligned} 15 - 4I_1 + 10I_3 - 20 &= 0 \\ 20 - 10I_3 - 5(I_1 + I_3) + 40 &= 0 \end{aligned}\right\} \text{Substituting for } I_2 \text{ (since it occurs only once in the two equations)}$$

or

$$\begin{aligned} -4I_1 + 10I_3 &= 5 \\ -5I_1 - 15I_3 &= -60 \end{aligned}$$

Multiplying the lower equation by -1, we have

$$\begin{aligned} -4I_1 + 10I_3 &= 5 \\ 5I_1 + 15I_3 &= 60 \end{aligned}$$

$$I_1 = \frac{\begin{vmatrix} 5 & 10 \\ 60 & 15 \end{vmatrix}}{\begin{vmatrix} -4 & 10 \\ 5 & 15 \end{vmatrix}} = \frac{75 - 600}{-60 - 50} = \frac{-525}{-110} = \mathbf{4.773\ A}$$

$$I_3 = \frac{\begin{vmatrix} -4 & 5 \\ 5 & 60 \end{vmatrix}}{-110} = \frac{-240 - 25}{-110} = \frac{-265}{-110} = \mathbf{2.409\ A}$$

$$I_2 = I_1 + I_3 = 4.773 + 2.409 = \mathbf{7.182\ A}$$

revealing that the assumed directions were the actual directions, with I_2 equal to the sum of I_1 and I_3.

8.7 MESH ANALYSIS (GENERAL APPROACH)

The second method of analysis to be described is called **mesh analysis.** The term *mesh* is derived from the similarities in appearance between the closed loops of a network and a wire mesh fence. Although this approach is on a more sophisticated plane than the branch-current method, it incorporates many of the ideas just developed. Of the two methods, mesh analysis is the one more frequently applied today. Branch-current analysis is introduced as a stepping stone to mesh analysis because branch currents are initially more "real" to the student than the **mesh (loop) currents** employed in mesh analysis. Essentially, the mesh-analysis approach simply eliminates the need to substitute the results of Kirchhoff's current law into the equations derived from Kirchhoff's voltage law. It is now accomplished in the initial writing of the equations. The systematic approach outlined below should be followed when applying this method.

1. ***Assign a distinct current in the clockwise direction to each independent, closed loop of the network. It is not absolutely necessary to choose the clockwise direction for each loop current. In fact, any direction can be chosen for each loop current with no loss in accuracy, as long as the remaining steps are followed properly. However, by choosing the clockwise direction as a standard, we can develop a shorthand method (Section 8.8) for writing the required equations that will save time and possibly prevent some common errors.***

This first step is accomplished most effectively by placing a loop current *within* each "window" of the network, as demonstrated in the previous section, to ensure that they are all independent. A variety of other loop currents can be assigned. In each case, however, be sure that the information carried by any one loop equation is not included in a combination of the other network equations. This is the crux of the terminology: *independent*. No matter how you choose your loop currents, the number of loop currents required is always equal to the number of windows of a planar (no-crossovers) network. On occasion a network may appear to be nonplanar. However, a redrawing of the network may reveal that it is, in fact, planar. Such may be the case in one or two problems at the end of the chapter.

Before continuing to the next step, let us ensure that the concept of a loop current is clear. For the network of Fig. 8.26, the loop current I_1 is the branch current of the branch containing the 2-Ω resistor and 2-V battery. The current through the 4-Ω resistor is not I_1, however, since there is also a loop current I_2 through it. Since they have opposite directions, $I_{4\Omega}$ equals the difference between the two, $I_1 - I_2$ or $I_2 - I_1$, depending on which you choose to be the defining direction. In other words, *a loop current is a branch current only when it is the only loop current assigned to that branch.*

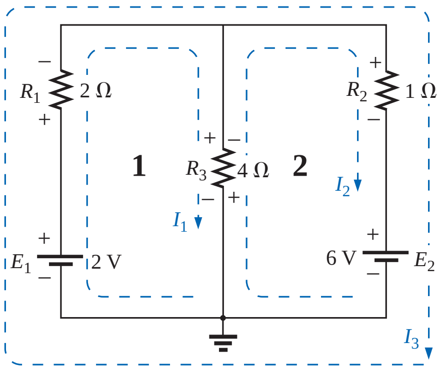


FIG. 8.26
Defining the mesh currents for a "two-window" network.

2. ***Indicate the polarities within each loop for each resistor as determined by the assumed direction of loop current for that loop. Note the requirement that the polarities be placed within each loop. This requires, as shown in Fig. 8.26, that the 4-Ω resistor have two sets of polarities across it.***
3. ***Apply Kirchhoff's voltage law around each closed loop in the clockwise direction. Again, the clockwise direction was chosen to establish uniformity and prepare us for the method to be introduced in the next section.***
   a. ***If a resistor has two or more assumed currents through it, the total current through the resistor is the assumed current of the loop in which Kirchhoff's voltage law is being applied, plus the assumed currents of the other loops passing through in the same direction, minus the assumed currents through in the opposite direction.***
   b. ***The polarity of a voltage source is unaffected by the direction of the assigned loop currents.***
4. ***Solve the resulting simultaneous linear equations for the assumed loop currents.***

EXAMPLE 8.11 Consider the same basic network as in Example 8.9 of the preceding section, now appearing in Fig. 8.26.

Solution:

Step 1: Two loop currents (I_1 and I_2) are assigned in the clockwise direction in the windows of the network. A third loop (I_3) could have been included around the entire network, but the information carried by this loop is already included in the other two.

Step 2: Polarities are drawn within each window to agree with assumed current directions. Note that for this case, the polarities across the 4-Ω resistor are the opposite for each loop current.

Step 3: Kirchhoff's voltage law is applied around each loop in the clockwise direction. Keep in mind as this step is performed that the law is concerned only with the magnitude and polarity of the voltages around the closed loop and not with whether a voltage rise or drop is due to a battery or a resistive element. The voltage across each resistor is determined by $V = IR$, and for a resistor with more than one current through it, the current is the loop current of the loop being examined plus or minus the other loop currents as determined by their directions. If clockwise applications of Kirchhoff's voltage law are always chosen, the other loop currents will always be subtracted from the loop current of the loop being analyzed.

loop 1: $+E_1 - V_1 - V_3 = 0$ (clockwise starting at point a)

$$+2\text{ V} - (2\ \Omega)\,I_1 - \overbrace{(4\ \Omega)\underbrace{(I_1 - I_2)}_{\substack{\text{Total current}\\ \text{through}\\ \text{4-}\Omega\text{ resistor}}}}^{\substack{\text{Voltage drop across}\\ \text{4-}\Omega\text{ resistor}}} = 0$$

Subtracted since I_2 is opposite in direction to I_1.

loop 2: $-V_3 - V_2 - E_2 = 0$ (clockwise starting at point b)

$$-(4\ \Omega)(I_2 - I_1) - (1\ \Omega)I_2 - 6\text{ V} = 0$$

Step 4: The equations are then rewritten as follows (without units for clarity):

$$\begin{aligned} \text{loop 1:}\quad & +2 - 2I_1 - 4I_1 + 4I_2 = 0 \\ \text{loop 2:}\quad & -4I_2 + 4I_1 - 1I_2 - 6 = 0 \end{aligned}$$

and

$$\begin{aligned} \text{loop 1:}\quad & +2 - 6I_1 + 4I_2 = 0 \\ \text{loop 2:}\quad & -5I_2 + 4I_1 - 6 = 0 \end{aligned}$$

or

$$\begin{aligned} \text{loop 1:}\quad & -6I_1 + 4I_2 = -2 \\ \text{loop 2:}\quad & +4I_1 - 5I_2 = +6 \end{aligned}$$

Applying determinants will result in

$$I_1 = \mathbf{-1\ A} \quad \text{and} \quad I_2 = \mathbf{-2\ A}$$

The minus signs indicate that the currents have a direction opposite to that indicated by the assumed loop current.

The actual current through the 2-V source and 2-Ω resistor is therefore 1 A in the other direction, and the current through the 6-V source and 1-Ω resistor is 2 A in the opposite direction indicated on the circuit. The current through the 4-Ω resistor is determined by the following equation from the original network:

$$\text{loop 1:}\quad I_{4\Omega} = I_1 - I_2 = -1\text{ A} - (-2\text{ A}) = -1\text{ A} + 2\text{ A} = \mathbf{1\text{ A}} \quad (\text{in the direction of } I_1)$$

The outer loop (I_3) and *one* inner loop (either I_1 or I_2) would also have produced the correct results. This approach, however, will often lead to errors since the loop equations may be more difficult to write. The best method of picking these loop currents is to use the window approach.

EXAMPLE 8.12 Find the current through each branch of the network of Fig. 8.27.

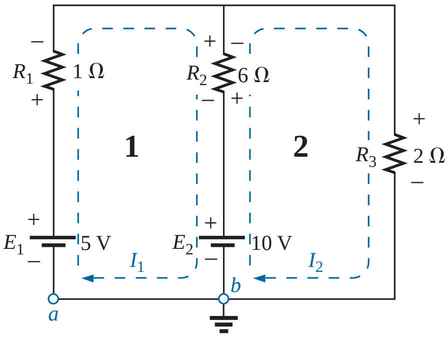


FIG. 8.27
Example 8.12.

Solution:

Steps 1 and 2 are as indicated in the circuit. Note that the polarities of the 6-Ω resistor are different for each loop current.

Step 3: Kirchhoff's voltage law is applied around each closed loop in the clockwise direction:

$$\text{loop 1:}\quad +E_1 - V_1 - V_2 - E_2 = 0 \quad (\text{clockwise starting at point } a)$$

$$+5\text{ V} - (1\ \Omega)I_1 - (6\ \Omega)(I_1 - I_2) - 10\text{ V} = 0$$

I_2 flows through the 6-Ω resistor in the direction opposite to I_1.

$$\text{loop 2:}\quad E_2 - V_2 - V_3 = 0 \quad (\text{clockwise starting at point } b)$$

$$+10\text{ V} - (6\ \Omega)(I_2 - I_1) - (2\ \Omega)I_2 = 0$$

The equations are rewritten as

$$\left.\begin{aligned} 5 - I_1 - 6I_1 + 6I_2 - 10 &= 0 \\ 10 - 6I_2 + 6I_1 - 2I_2 &= 0 \end{aligned}\right\} \begin{aligned} -7I_1 + 6I_2 &= 5 \\ +6I_1 - 8I_2 &= -10 \end{aligned}$$

Step 4:

$$I_1 = \frac{\begin{vmatrix} 5 & 6 \\ -10 & -8 \end{vmatrix}}{\begin{vmatrix} -7 & 6 \\ 6 & -8 \end{vmatrix}} = \frac{-40 + 60}{56 - 36} = \frac{20}{20} = \mathbf{1\text{ A}}$$

$$I_2 = \frac{\begin{vmatrix} -7 & 5 \\ 6 & -10 \end{vmatrix}}{20} = \frac{70 - 30}{20} = \frac{40}{20} = \mathbf{2\text{ A}}$$

Since I_1 and I_2 are positive and flow in opposite directions through the 6-Ω resistor and 10-V source, the total current in this branch is equal to the difference of the two currents in the direction of the larger:

$$I_2 > I_1 \quad (2\text{ A} > 1\text{ A})$$

Therefore,

$$I_{R_2} = I_2 - I_1 = 2\text{ A} - 1\text{ A} = \mathbf{1\text{ A}} \quad \text{in the direction of } I_2$$

It is sometimes impractical to draw all the branches of a circuit at right angles to one another. The next example demonstrates how a portion of a network may appear due to various constraints. The method of analysis does not change with this change in configuration.

EXAMPLE 8.13 Find the branch currents of the network of Fig. 8.28.

Solution:

Steps 1 and 2 are as indicated in the circuit.

Step 3: Kirchhoff's voltage law is applied around each closed loop:

$$\text{loop 1:}\quad -E_1 - I_1R_1 - E_2 - V_2 = 0 \quad \text{(clockwise from point } a\text{)}$$
$$-6\text{ V} - (2\ \Omega)I_1 - 4\text{ V} - (4\ \Omega)(I_1 - I_2) = 0$$
$$\text{loop 2:}\quad -V_2 + E_2 - V_3 - E_3 = 0 \quad \text{(clockwise from point } b\text{)}$$
$$-(4\ \Omega)(I_2 - I_1) + 4\text{ V} - (6\ \Omega)(I_2) - 3\text{ V} = 0$$

which are rewritten as

$$\left.\begin{aligned} -10 - 4I_1 - 2I_1 + 4I_2 &= 0 \\ +\ 1 + 4I_1 - 4I_2 - 6I_2 &= 0 \end{aligned}\right\} \begin{aligned} -6I_1 + \ \ 4I_2 &= +10 \\ +4I_1 - 10I_2 &= -1 \end{aligned}$$

or, by multiplying the top equation by -1, we obtain

$$\begin{aligned} 6I_1 - \ \ 4I_2 &= -10 \\ 4I_1 - 10I_2 &= -1 \end{aligned}$$

Step 4:

$$I_1 = \frac{\begin{vmatrix} -10 & -4 \\ -1 & -10 \end{vmatrix}}{\begin{vmatrix} 6 & -4 \\ 4 & -10 \end{vmatrix}} = \frac{100 - 4}{-60 + 16} = \frac{96}{-44} = \mathbf{-2.182\ A}$$

Using the TI-86 calculator:

det[[−10,−4][−1,−10]]/det[[6,−4][4,−10]] (ENTER)	−2.182

CALC. 8.2

$$I_2 = \frac{\begin{vmatrix} 6 & -10 \\ 4 & -1 \end{vmatrix}}{-44} = \frac{-6 + 40}{-44} = \frac{34}{-44} = \mathbf{-0.773\ A}$$

The current in the 4-Ω resistor and 4-V source for loop 1 is

$$\begin{aligned} I_1 - I_2 &= -2.182\text{ A} - (-0.773\text{ A}) \\ &= -2.182\text{ A} + 0.773\text{ A} \\ &= \mathbf{-1.409\ A} \end{aligned}$$

revealing that it is 1.409 A in a direction opposite (due to the minus sign) to I_1 in loop 1.

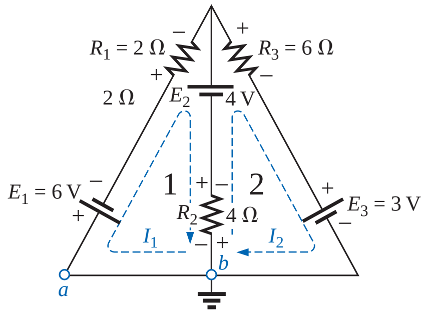


FIG. 8.28
Example 8.13.

Supermesh Currents

On occasion there will be current sources in the network to which mesh analysis is to be applied. In such cases one can convert the current source to a voltage source (if a parallel resistor is present) and proceed as before or utilize a *supermesh* current and proceed as follows.

Start as before and assign a mesh current to each independent loop, including the current sources, as if they were resistors or voltage sources. Then mentally (redraw the network if necessary) remove the current sources (replace with open-circuit equivalents), and apply

Kirchhoff's voltage law to all the remaining independent paths of the network using the mesh currents just defined. Any resulting path, including two or more mesh currents, is said to be the path of a *supermesh* current. Then relate the chosen mesh currents of the network to the independent current sources of the network, and solve for the mesh currents. The next example will clarify the definition of a *supermesh* current and the procedure.

EXAMPLE 8.14 Using mesh analysis, determine the currents of the network of Fig. 8.29.

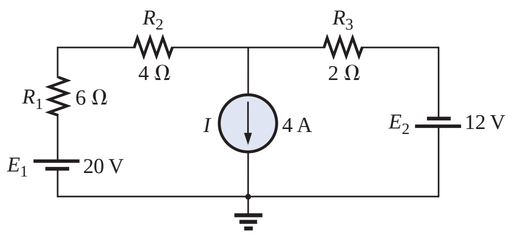


FIG. 8.29
Example 8.14.

Solution: First, the mesh currents for the network are defined, as shown in Fig. 8.30. Then the current source is mentally removed, as shown in Fig. 8.31, and Kirchhoff's voltage law is applied to the resulting network. The single path now including the effects of two mesh currents is referred to as the path of a *supermesh* current.

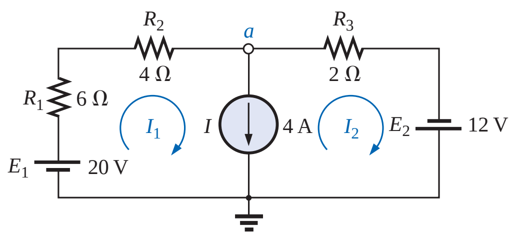


FIG. 8.30
Defining the mesh currents for the network of Fig. 8.29.

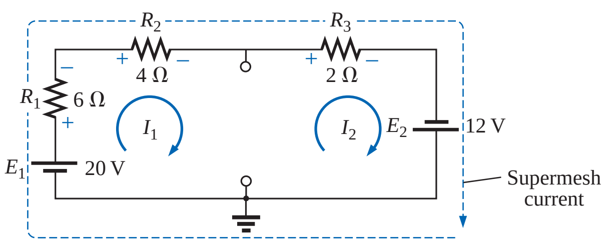


FIG. 8.31
Defining the supermesh current.

Applying Kirchhoff's law:

$$20\text{ V} - I_1(6\ \Omega) - I_1(4\ \Omega) - I_2(2\ \Omega) + 12\text{ V} = 0$$

or

$$10I_1 + 2I_2 = 32$$

Node a is then used to relate the mesh currents and the current source using Kirchhoff's current law:

$$I_1 = I + I_2$$

The result is two equations and two unknowns:

$$\begin{aligned} 10I_1 + 2I_2 &= 32 \\ I_1 - \quad I_2 &= 4 \end{aligned}$$

Applying determinants:

$$I_1 = \frac{\begin{vmatrix} 32 & 2 \\ 4 & -1 \end{vmatrix}}{\begin{vmatrix} 10 & 2 \\ 1 & -1 \end{vmatrix}} = \frac{(32)(-1) - (2)(4)}{(10)(-1) - (2)(1)} = \frac{40}{12} = \mathbf{3.33\ A}$$

and $$I_2 = I_1 - I = 3.33\text{ A} - 4\text{ A} = \mathbf{-0.67\ A}$$

In the above analysis, it might appear that when the current source was removed, $I_1 = I_2$. However, the supermesh approach requires that we stick with the original definition of each mesh current and not alter those definitions when current sources are removed.

EXAMPLE 8.15 Using mesh analysis, determine the currents for the network of Fig. 8.32.

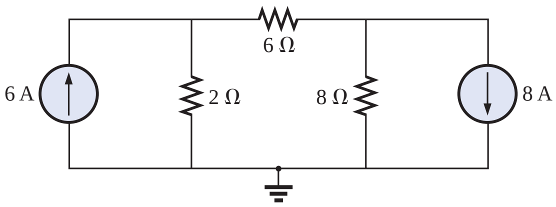


FIG. 8.32
Example 8.15.

Solution: The mesh currents are defined in Fig. 8.33. The current sources are removed, and the single supermesh path is defined in Fig. 8.34.

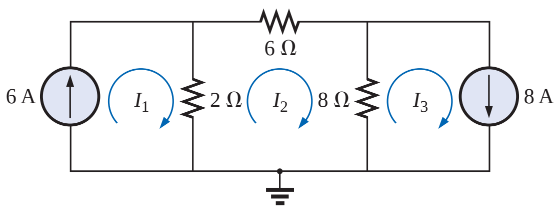


FIG. 8.33
Defining the mesh currents for the network of Fig. 8.32.

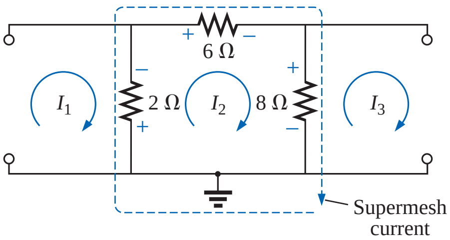


FIG. 8.34
Defining the supermesh current for the network of Fig. 8.32.

Applying Kirchhoff's voltage law around the supermesh path:

$$\begin{aligned} &-V_{2\Omega} - V_{6\Omega} - V_{8\Omega} = 0 \\ &-(I_2 - I_1)2\ \Omega - I_2(6\ \Omega) - (I_2 - I_3)8\ \Omega = 0 \\ &-2I_2 + 2I_1 - 6I_2 - 8I_2 + 8I_3 = 0 \\ &\ \ 2I_1 - 16I_2 + 8I_3 = 0 \end{aligned}$$

Introducing the relationship between the mesh currents and the current sources:

$$I_1 = 6\text{ A}$$
$$I_3 = 8\text{ A}$$

results in the following solutions:

$$2I_1 - 16I_2 + 8I_3 = 0$$
$$2(6\text{ A}) - 16I_2 + 8(8\text{ A}) = 0$$

and $$I_2 = \frac{76\text{ A}}{16} = \mathbf{4.75\text{ A}}$$

Then $$I_{2\Omega}\downarrow = I_1 - I_2 = 6\text{ A} - 4.75\text{ A} = \mathbf{1.25\text{ A}}$$

and $$I_{8\Omega}\uparrow = I_3 - I_2 = 8\text{ A} - 4.75\text{ A} = \mathbf{3.25\text{ A}}$$

Again, note that you must stick with your original definitions of the various mesh currents when applying Kirchhoff's voltage law around the resulting supermesh paths.

8.8 MESH ANALYSIS (FORMAT APPROACH)

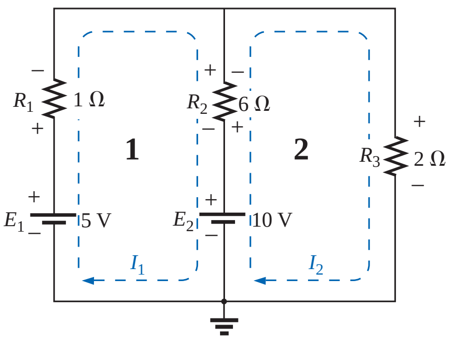


FIG. 8.35
Network of Fig. 8.27 redrawn with assigned loop currents.

Now that the basis for the mesh-analysis approach has been established, we will now examine a technique for writing the mesh equations more rapidly and usually with fewer errors. As an aid in introducing the procedure, the network of Example 8.12 (Fig. 8.27) has been redrawn in Fig. 8.35 with the assigned loop currents. (Note that each loop current has a clockwise direction.)

The equations obtained are

$$-7I_1 + 6I_2 = 5$$
$$6I_1 - 8I_2 = -10$$

which can also be written as

$$7I_1 - 6I_2 = -5$$
$$8I_2 - 6I_1 = 10$$

and expanded as

Col. 1	Col. 2	Col. 3
$(1 + 6)I_1 -$	$6I_2$	$= (5 - 10)$
$(2 + 6)I_2 -$	$6I_1$	$= 10$

Note in the above equations that column 1 is composed of a loop current times the sum of the resistors through which that loop current passes. Column 2 is the product of the resistors common to another loop current times that other loop current. Note that in each equation, this column is subtracted from column 1. Column 3 is the *algebraic* sum of the voltage sources through which the loop current of interest passes. A source is assigned a positive sign if the loop current passes from the negative to the positive terminal, and a negative value is assigned if the polarities are reversed. The comments above are correct only for a standard direction of loop current in each window, the one chosen being the clockwise direction.

The above statements can be extended to develop the following *format approach* to mesh analysis:

1. ***Assign a loop current to each independent, closed loop (as in the previous section) in a clockwise direction.***
2. ***The number of required equations is equal to the number of chosen independent, closed loops. Column 1 of each equation is formed by summing the resistance values of those resistors through which the loop current of interest passes and multiplying the result by that loop current.***
3. ***We must now consider the mutual terms, which, as noted in the examples above, are always subtracted from the first column. A* mutual term *is simply any resistive element having an additional loop current passing through it. It is possible to have more than one mutual term if the loop current of interest has an element in common with more than one other loop current. This will be demonstrated in an example to follow. Each term is the product of the mutual resistor and the other loop current passing through the same element.***
4. ***The column to the right of the equality sign is the algebraic sum of the voltage sources through which the loop current of interest passes. Positive signs are assigned to those sources of voltage having a polarity such that the loop current passes from the negative to the positive terminal. A negative sign is assigned to those potentials for which the reverse is true.***
5. ***Solve the resulting simultaneous equations for the desired loop currents.***

Before considering a few examples, be aware that since the column to the right of the equals sign is the algebraic sum of the voltage sources in that loop, *the format approach can be applied only to networks in which all current sources have been converted to their equivalent voltage source.*

EXAMPLE 8.16 Write the mesh equations for the network of Fig. 8.36, and find the current through the 7-Ω resistor.

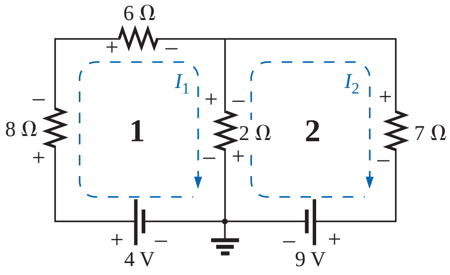


FIG. 8.36
Example 8.16.

Solution:

Step 1: As indicated in Fig. 8.36, each assigned loop current has a clockwise direction.

Steps 2 to 4:

$$I_1: \quad (8\ \Omega + 6\ \Omega + 2\ \Omega)I_1 - (2\ \Omega)I_2 = 4\ \text{V}$$
$$I_2: \quad (7\ \Omega + 2\ \Omega)I_2 - (2\ \Omega)I_1 = -9\ \text{V}$$

and

$$16I_1 - 2I_2 = 4$$
$$9I_2 - 2I_1 = -9$$

which, for determinants, are

$$16I_1 - 2I_2 = 4$$
$$-2I_1 + 9I_2 = -9$$

and

$$I_2 = I_{7\Omega} = \frac{\begin{vmatrix} 16 & 4 \\ -2 & -9 \end{vmatrix}}{\begin{vmatrix} 16 & -2 \\ -2 & 9 \end{vmatrix}} = \frac{-144 + 8}{144 - 4} = \frac{-136}{140}$$
$$= \mathbf{-0.971\ A}$$

EXAMPLE 8.17 Write the mesh equations for the network of Fig. 8.37.

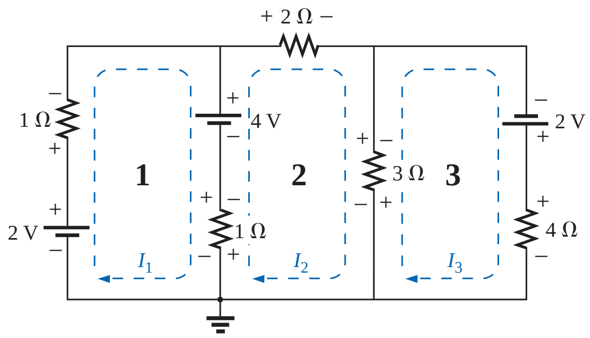


FIG. 8.37
Example 8.17.

Solution:

Step 1: Each window is assigned a loop current in the clockwise direction:

I_1 does not pass through an element mutual with I_3.

$$
\begin{aligned}
I_1:&\quad (1\,\Omega + 1\,\Omega)I_1 - (1\,\Omega)I_2 + 0 = 2\text{ V} - 4\text{ V}\\
I_2:&\quad (1\,\Omega + 2\,\Omega + 3\,\Omega)I_2 - (1\,\Omega)I_1 - (3\,\Omega)I_3 = 4\text{ V}\\
I_3:&\quad (3\,\Omega + 4\,\Omega)I_3 - (3\,\Omega)I_2 + 0 = 2\text{ V}
\end{aligned}
$$

I_3 does not pass through an element mutual with I_1.

Summing terms yields

$$
\begin{aligned}
2I_1 - I_2 + 0 &= -2\\
6I_2 - I_1 - 3I_3 &= 4\\
7I_3 - 3I_2 + 0 &= 2
\end{aligned}
$$

which are rewritten for determinants as

$$
\begin{array}{c|cccccc}
 & c & & b & & a & \\
 & 2I_1 & - & I_2 & + & 0 & = -2\\
b & -I_1 & + & 6I_2 & - & 3I_3 & = 4\\
a & 0 & - & 3I_2 & + & 7I_3 & = 2
\end{array}
$$

Note that the coefficients of the *a* and *b* diagonals are equal. This *symmetry* about the *c*-axis will always be true for equations written using the format approach. It is a check on whether the equations were obtained correctly.

We will now consider a network with only one source of voltage to point out that mesh analysis can be used to advantage in other than multisource networks.

EXAMPLE 8.18 Find the current through the 10-Ω resistor of the network of Fig. 8.38.

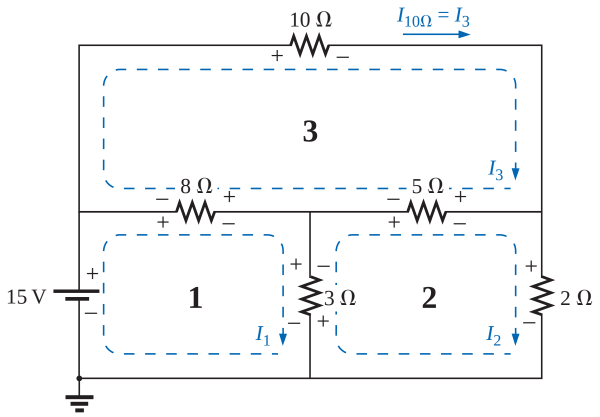


FIG. 8.38
Example 8.18.

Solution 1:

$$
\begin{aligned}
I_1&: \quad (8\ \Omega + 3\ \Omega)I_1 - (8\ \Omega)I_3 - (3\ \Omega)I_2 = 15\ \text{V} \\
I_2&: \quad (3\ \Omega + 5\ \Omega + 2\ \Omega)I_2 - (3\ \Omega)I_1 - (5\ \Omega)I_3 = 0 \\
I_3&: \quad (8\ \Omega + 10\ \Omega + 5\ \Omega)I_3 - (8\ \Omega)I_1 - (5\ \Omega)I_2 = 0
\end{aligned}
$$

$$
\begin{aligned}
11I_1 - 8I_3 - 3I_2 &= 15 \\
10I_2 - 3I_1 - 5I_3 &= 0 \\
23I_3 - 8I_1 - 5I_2 &= 0
\end{aligned}
$$

or

$$
\begin{aligned}
11I_1 - 3I_2 - 8I_3 &= 15 \\
-3I_1 + 10I_2 - 5I_3 &= 0 \\
-8I_1 - 5I_2 + 23I_3 &= 0
\end{aligned}
$$

and

$$
I_3 = I_{10\Omega} = \frac{\begin{vmatrix} 11 & -3 & 15 \\ -3 & 10 & 0 \\ -8 & -5 & 0 \end{vmatrix}}{\begin{vmatrix} 11 & -3 & -8 \\ -3 & 10 & -5 \\ -8 & -5 & 23 \end{vmatrix}} = \mathbf{1.220\ A}
$$

Mathcad Solution: For this example, rather than take the time to develop the determinant form for each variable, we will apply Mathcad directly to the resulting equations. As shown in Fig. 8.39, a **Guess** value for each variable must first be defined. Such guessing helps the computer begin its iteration process as it searches for the solution. By providing a rough estimate of 1, the computer recognizes that the result will probably be a number with a magnitude less than 100 rather than have to worry about solutions that extend into the thousands or tens of thousands—the search has been narrowed considerably.

Next, as shown, the word **Given** must be entered to tell the computer that the defining equations will follow. Finally, each equation must be carefully entered and set equal to the constant on the right using the **Ctrl=** operation.

The results are then obtained with the **Find(I1,I2,I3)** expression and an equal sign. As shown, the results are available with an acceptable degree of accuracy even though entering the equations and performing the analysis took only a minute or two (with practice).

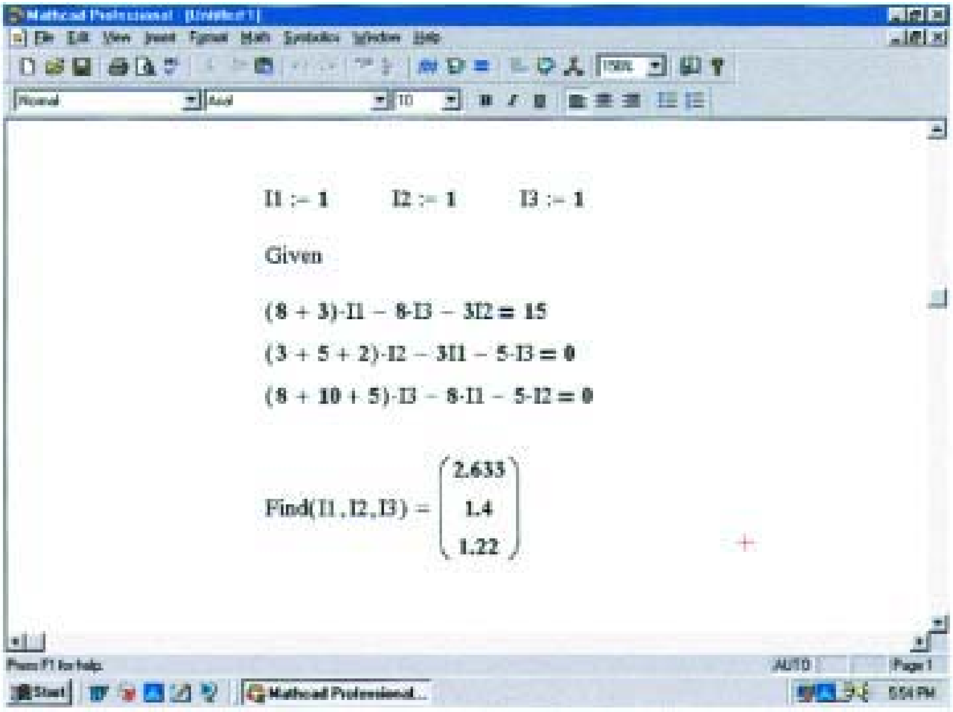


FIG. 8.39
Using Mathcad to verify the numerical calculations of Example 8.18.

Solution 2: Using the TI-86 calculator:

det[[11, −3,15][−3,10,0][−8, −5,0]]/det[[11, −3, −8][−3,10, −5][−8, −5,23]] ENTER	1.220

CALC. 8.3

This display certainly requires some care in entering the correct sequence of brackets in the required format, but it is still a rather neat, compact format.

8.9 NODAL ANALYSIS (GENERAL APPROACH)

Recall from the development of loop analysis that the general network equations were obtained by applying Kirchhoff's voltage law around each closed loop. We will now employ Kirchhoff's current law to develop a method referred to as **nodal analysis.**

A **node** is defined as a junction of two or more branches. If we now define one node of any network as a reference (that is, a point of zero potential or ground), the remaining nodes of the network will all have a fixed potential relative to this reference. For a network of N nodes, therefore, there will exist $(N - 1)$ nodes with a fixed potential relative to the assigned reference node. Equations relating these nodal voltages can be written by applying Kirchhoff's current law at each of the $(N - 1)$ nodes. To obtain the complete solution of a network, these nodal voltages are then evaluated in the same manner in which loop currents were found in loop analysis.

The nodal analysis method is applied as follows:

1. ***Determine the number of nodes within the network.***
2. ***Pick a reference node, and label each remaining node with a subscripted value of voltage: V_1, V_2, and so on.***
3. ***Apply Kirchhoff's current law at each node except the reference. Assume that all unknown currents leave the node for each application of Kirchhoff's current law. In other words, for each node, don't be influenced by the direction that an unknown current for another node may have had. Each node is to be treated as a separate entity, independent of the application of Kirchhoff's current law to the other nodes.***
4. ***Solve the resulting equations for the nodal voltages.***

A few examples will clarify the procedure defined by step 3. It will initially take some practice writing the equations for Kirchhoff's current law correctly, but in time the advantage of assuming that all the currents leave a node rather than identifying a specific direction for each branch will become obvious. (The same type of advantage is associated with assuming that all the mesh currents are clockwise when applying mesh analysis.)

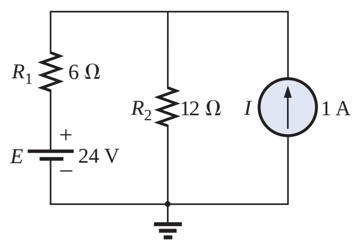


FIG. 8.40
Example 8.19.

EXAMPLE 8.19 Apply nodal analysis to the network of Fig. 8.40.

Solution:

Steps 1 and 2: The network has two nodes, as shown in Fig. 8.41. The lower node is defined as the reference node at ground potential (zero volts), and the other node as V_1, the voltage from node 1 to ground.

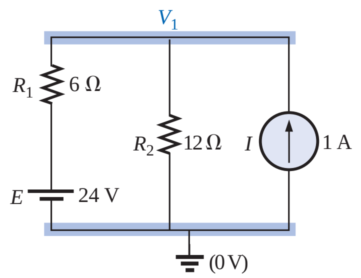


FIG. 8.41
Network of Fig. 8.40 with assigned nodes.

Step 3: I_1 and I_2 are defined as leaving the node in Fig. 8.42, and Kirchhoff's current law is applied as follows:

$$I = I_1 + I_2$$

The current I_2 is related to the nodal voltage V_1 by Ohm's law:

$$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_1}{R_2}$$

The current I_1 is also determined by Ohm's law as follows:

$$I_1 = \frac{V_{R_1}}{R_1}$$

with $$V_{R_1} = V_1 - E$$

Substituting into the Kirchhoff's current law equation:

$$I = \frac{V_1 - E}{R_1} + \frac{V_1}{R_2}$$

and rearranging, we have

$$I = \frac{V_1}{R_1} - \frac{E}{R_1} + \frac{V_1}{R_2} = V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{E}{R_1}$$

or $$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{E}{R_1} + I$$

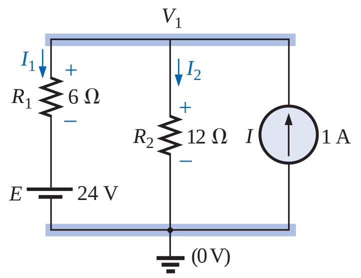


FIG. 8.42
Applying Kirchhoff's current law to the node V_1.

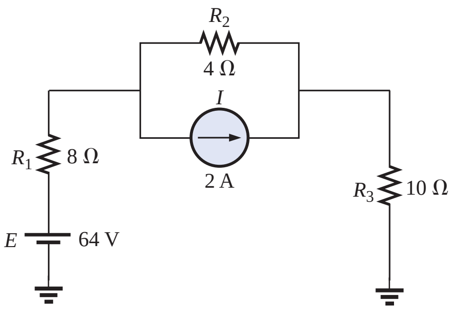


FIG. 8.43
Example 8.20.

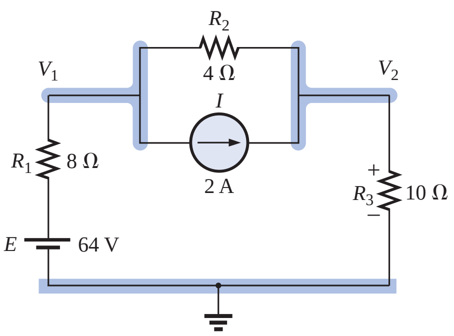


FIG. 8.44
Defining the nodes for the network of Fig. 8.43.

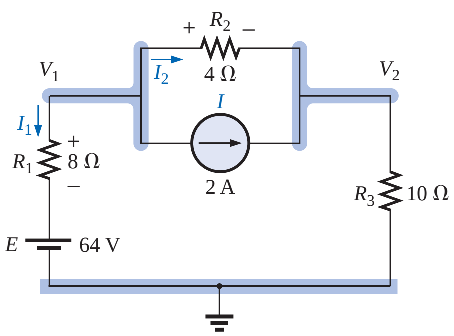


FIG. 8.45
Applying Kirchhoff's current law to node V_1.

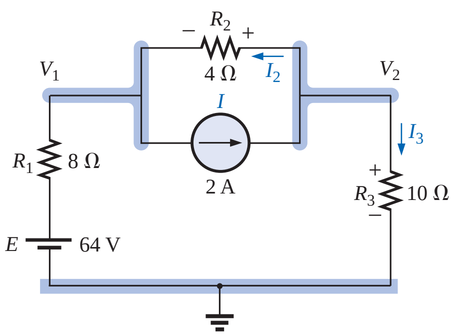


FIG. 8.46
Applying Kirchhoff's current law to node V_2.

Substituting numerical values, we obtain

$$V_1\left(\frac{1}{6\ \Omega} + \frac{1}{12\ \Omega}\right) = \frac{24\ \text{V}}{6\ \Omega} + 1\ \text{A} = 4\ \text{A} + 1\ \text{A}$$

$$V_1\left(\frac{1}{4\ \Omega}\right) = 5\ \text{A}$$

$$V_1 = \mathbf{20\ V}$$

The currents I_1 and I_2 can then be determined using the preceding equations:

$$I_1 = \frac{V_1 - E}{R_1} = \frac{20\ \text{V} - 24\ \text{V}}{6\ \Omega} = \frac{-4\ \text{V}}{6\ \Omega}$$

$$= \mathbf{-0.667\ A}$$

The minus sign indicates simply that the current I_1 has a direction opposite to that appearing in Fig. 8.42.

$$I_2 = \frac{V_1}{R_2} = \frac{20\ \text{V}}{12\ \Omega} = \mathbf{1.667\ A}$$

EXAMPLE 8.20 Apply nodal analysis to the network of Fig. 8.43.

Solution 1:

Steps 1 and 2: The network has three nodes, as defined in Fig. 8.44, with the bottom node again defined as the reference node (at ground potential, or zero volts), and the other nodes as V_1 and V_2.

Step 3: For node V_1 the currents are defined as shown in Fig. 8.45, and Kirchhoff's current law is applied:

$$0 = I_1 + I_2 + I$$

with
$$I_1 = \frac{V_1 - E}{R_1}$$

and
$$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_1 - V_2}{R_2}$$

so that
$$\frac{V_1 - E}{R_1} + \frac{V_1 - V_2}{R_2} + I = 0$$

or
$$\frac{V_1}{R_1} - \frac{E}{R_1} + \frac{V_1}{R_2} - \frac{V_2}{R_2} + I = 0$$

and
$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{R_2}\right) = -I + \frac{E}{R_1}$$

Substituting values:

$$V_1\left(\frac{1}{8\ \Omega} + \frac{1}{4\ \Omega}\right) - V_2\left(\frac{1}{4\ \Omega}\right) = -2\ \text{A} + \frac{64\ \text{V}}{8\ \Omega} = 6\ \text{A}$$

For node V_2 the currents are defined as shown in Fig. 8.46, and Kirchhoff's current law is applied:

$$I = I_2 + I_3$$

with
$$I = \frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3}$$

or
$$I = \frac{V_2}{R_2} - \frac{V_1}{R_2} + \frac{V_2}{R_3}$$

and
$$V_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - V_1\left(\frac{1}{R_2}\right) = I$$

Substituting values:

$$V_2\left(\frac{1}{4\,\Omega} + \frac{1}{10\,\Omega}\right) - V_1\left(\frac{1}{4\,\Omega}\right) = 2\text{ A}$$

Step 4: The result is two equations and two unknowns:

$$V_1\left(\frac{1}{8\,\Omega} + \frac{1}{4\,\Omega}\right) - V_2\left(\frac{1}{4\,\Omega}\right) = 6\text{ A}$$
$$-V_1\left(\frac{1}{4\,\Omega}\right) + V_2\left(\frac{1}{4\,\Omega} + \frac{1}{10\,\Omega}\right) = 2\text{ A}$$

which become

$$\begin{aligned} 0.375V_1 - 0.25V_2 &= 6 \\ -0.25V_1 + 0.35V_2 &= 2 \end{aligned}$$

Using determinants,

$$V_1 = \mathbf{37.818\text{ V}}$$
$$V_2 = \mathbf{32.727\text{ V}}$$

Since E is greater than V_1, the current I_1 flows from ground to V_1 and is equal to

$$I_{R_1} = \frac{E - V_1}{R_1} = \frac{64\text{ V} - 37.818\text{ V}}{8\,\Omega} = \mathbf{3.273\text{ A}}$$

The positive value for V_2 results in a current I_{R_3} from node V_2 to ground equal to

$$I_{R_3} = \frac{V_{R_3}}{R_3} = \frac{V_2}{R_3} = \frac{32.727\text{ V}}{10\,\Omega} = \mathbf{3.273\text{ A}}$$

Since V_1 is greater than V_2, the current I_{R_2} flows from V_1 to V_2 and is equal to

$$I_{R_2} = \frac{V_1 - V_2}{R_2} = \frac{37.818\text{ V} - 32.727\text{ V}}{4\,\Omega} = \mathbf{1.273\text{ A}}$$

Mathcad Solution: For this example, we will use Mathcad to work directly with the Kirchhoff's current law equations rather than taking the mathematical process down the line to more familiar forms. Simply define everything correctly, provide the **Guess** values, and insert **Given** where required. The process should be quite straightforward.

Note in Fig. 8.47 that the first equation comes from the fact that $I_1 + I_2 + I = 0$ while the second equation comes from $I_2 + I_3 = I$. Pay particular attention to the fact that the first equation is defined by Fig. 8.45 and the second by Fig. 8.46 because the direction of I_2 is different for each.

The results of $V_1 = 37.82$ V and $V_2 = 32.73$ V confirm the theoretical solution.

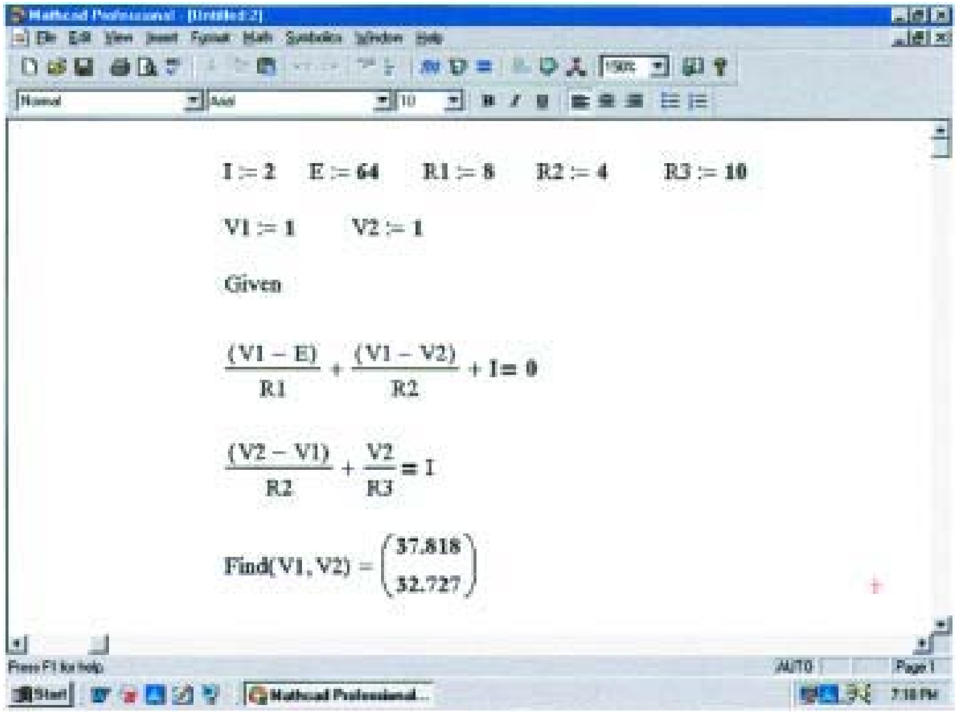


FIG. 8.47
Using Mathcad to verify the mathematical calculations of Example 8.20.

EXAMPLE 8.21 Determine the nodal voltages for the network of Fig. 8.48.

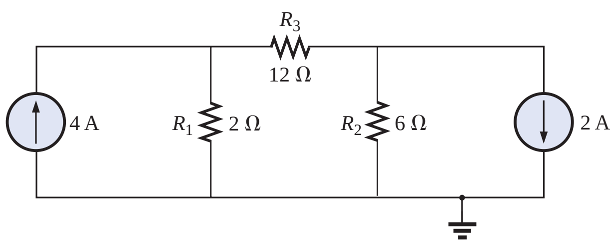


FIG. 8.48
Example 8.21.

Solution:

Steps 1 and 2: As indicated in Fig. 8.49.

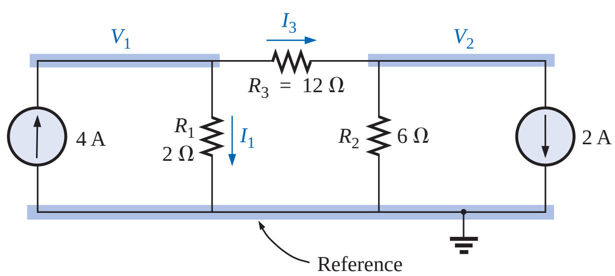


FIG. 8.49
Defining the nodes and applying Kirchhoff's current law to the node V_1.

Step 3: Included in Fig. 8.49 for the node V_1. Applying Kirchhoff's current law:

$$4\text{ A} = I_1 + I_3$$

and
$$4\text{ A} = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_3} = \frac{V_1}{2\,\Omega} + \frac{V_1 - V_2}{12\,\Omega}$$

Expanding and rearranging:

$$V_1\left(\frac{1}{2\,\Omega} + \frac{1}{12\,\Omega}\right) - V_2\left(\frac{1}{12\,\Omega}\right) = 4\text{ A}$$

For node V_2 the currents are defined as in Fig. 8.50.

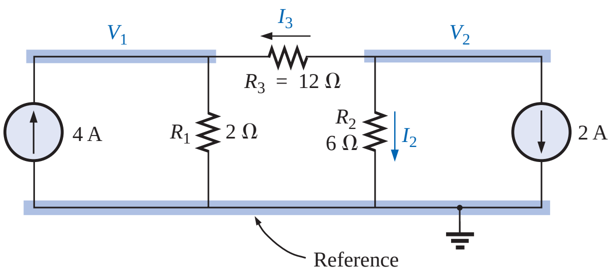


FIG. 8.50
Applying Kirchhoff's current law to the node V_2.

Applying Kirchhoff's current law:

$$0 = I_3 + I_2 + 2\text{ A}$$

and
$$\frac{V_2 - V_1}{R_3} + \frac{V_2}{R_2} + 2\text{ A} = 0 \longrightarrow \frac{V_2 - V_1}{12\,\Omega} + \frac{V_2}{6\,\Omega} + 2\text{ A} = 0$$

Expanding and rearranging:

$$V_2\left(\frac{1}{12\,\Omega} + \frac{1}{6\,\Omega}\right) - V_1\left(\frac{1}{12\,\Omega}\right) = -2\text{ A}$$

resulting in two equations and two unknowns (numbered for later reference):

$$\left.\begin{aligned} V_1\left(\frac{1}{2\,\Omega} + \frac{1}{12\,\Omega}\right) - V_2\left(\frac{1}{12\,\Omega}\right) &= +4\text{ A} \\ V_2\left(\frac{1}{12\,\Omega} + \frac{1}{6\,\Omega}\right) - V_1\left(\frac{1}{12\,\Omega}\right) &= -2\text{ A} \end{aligned}\right\} \qquad \textbf{(8.3)}$$

producing

$$\left.\begin{aligned} \frac{7}{12}V_1 - \frac{1}{12}V_2 &= +4 \\ -\frac{1}{12}V_1 + \frac{3}{12}V_2 &= -2 \end{aligned}\right\} \quad \begin{aligned} 7V_1 - V_2 &= 48 \\ -1V_1 + 3V_2 &= -24 \end{aligned}$$

and
$$V_1 = \frac{\begin{vmatrix} 48 & -1 \\ -24 & 3 \end{vmatrix}}{\begin{vmatrix} 7 & -1 \\ -1 & 3 \end{vmatrix}} = \frac{120}{20} = \mathbf{+6\text{ V}}$$

$$V_2 = \frac{\begin{vmatrix} 7 & 48 \\ -1 & -24 \end{vmatrix}}{20} = \frac{-120}{20} = \mathbf{-6\text{ V}}$$

Since V_1 is greater than V_2, the current through R_3 passes from V_1 to V_2. Its value is

$$I_{R_3} = \frac{V_1 - V_2}{R_3} = \frac{6\text{ V} - (-6\text{ V})}{12\ \Omega} = \frac{12\text{ V}}{12\ \Omega} = \mathbf{1\ A}$$

The fact that V_1 is positive results in a current I_{R_1} from V_1 to ground equal to

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{V_1}{R_1} = \frac{6\text{ V}}{2\ \Omega} = \mathbf{3\ A}$$

Finally, since V_2 is negative, the current I_{R_2} flows from ground to V_2 and is equal to

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_2}{R_2} = \frac{6\text{ V}}{6\ \Omega} = \mathbf{1\ A}$$

Supernode

On occasion there will be independent voltage sources in the network to which nodal analysis is to be applied. In such cases we can convert the voltage source to a current source (if a series resistor is present) and proceed as before, or we can introduce the concept of a *supernode* and proceed as follows.

Start as before and assign a nodal voltage to each independent node of the network, including each independent voltage source as if it were a resistor or current source. Then mentally replace the independent voltage sources with short-circuit equivalents, and apply Kirchhoff's current law to the defined nodes of the network. Any node including the effect of elements tied only to *other* nodes is referred to as a *supernode* (since it has an additional number of terms). Finally, relate the defined nodes to the independent voltage sources of the network, and solve for the nodal voltages. The next example will clarify the definition of *supernode.*

EXAMPLE 8.22 Determine the nodal voltages V_1 and V_2 of Fig. 8.51 using the concept of a supernode.

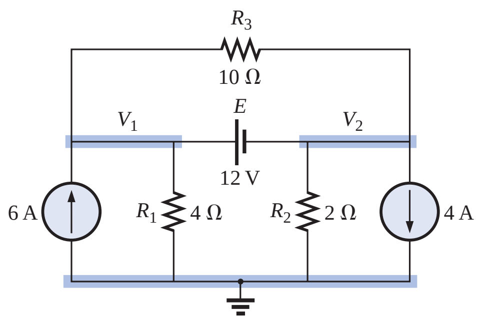


FIG. 8.51
Example 8.22.

Solution: Replacing the independent voltage source of 12 V with a short-circuit equivalent will result in the network of Fig. 8.52. Even though the mental application of a short-circuit equivalent is discussed above, it would be wise in the early stage of development to redraw the

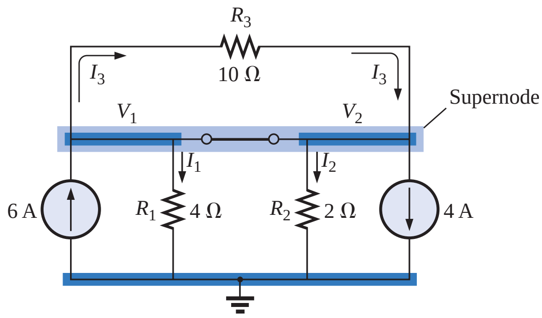


FIG. 8.52
Defining the supernode for the network of Fig. 8.51.

network as shown in Fig. 8.52. The result is a single supernode for which Kirchhoff's current law must be applied. Be sure to leave the other defined nodes in place and use them to define the currents from that region of the network. In particular, note that the current I_3 will leave the supernode at V_1 and then enter the same supernode at V_2. It must therefore appear twice when applying Kirchhoff's current law, as shown below:

$$\Sigma I_i = \Sigma I_o$$

$$6\text{ A} + I_3 = I_1 + I_2 + 4\text{ A} + I_3$$

or
$$I_1 + I_2 = 6\text{ A} - 4\text{ A} = 2\text{ A}$$

Then
$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = 2\text{ A}$$

and
$$\frac{V_1}{4\ \Omega} + \frac{V_2}{2\ \Omega} = 2\text{ A}$$

Relating the defined nodal voltages to the independent voltage source, we have

$$V_1 - V_2 = E = 12\text{ V}$$

which results in two equations and two unknowns:

$$\begin{aligned} 0.25V_1 + 0.5V_2 &= 2 \\ V_1 - \ \ 1V_2 &= 12 \end{aligned}$$

Substituting:

$$V_1 = V_2 + 12$$

$$0.25(V_2 + 12) + 0.5V_2 = 2$$

and
$$0.75V_2 = 2 - 3 = -1$$

so that
$$V_2 = \frac{-1}{0.75} = \mathbf{-1.333\ V}$$

and
$$V_1 = V_2 + 12\text{ V} = -1.333\text{ V} + 12\text{ V} = \mathbf{+10.667\ V}$$

The current of the network can then be determined as follows:

$$I_1\downarrow = \frac{V}{R_1} = \frac{10.667\text{ V}}{4\ \Omega} = \mathbf{2.667\ A}$$

$$I_2\uparrow = \frac{V_2}{R_2} = \frac{1.333\text{ V}}{2\ \Omega} = \mathbf{0.667\ A}$$

$$\underset{\rightarrow}{I_3} = \frac{V_1 - V_2}{10\ \Omega} = \frac{10.667\text{ V} - (-1.333\text{ V})}{10\ \Omega} = \frac{12\text{ V}}{10\ \Omega} = \mathbf{1.2\ A}$$

A careful examination of the network at the beginning of the analysis would have revealed that the voltage across the resistor R_3 must be 12 V and I_3 must be equal to 1.2 A.

8.10 NODAL ANALYSIS (FORMAT APPROACH)

A close examination of Eq. (8.3) appearing in Example 8.21 reveals that the subscripted voltage at the node in which Kirchhoff's current law is applied is multiplied by the sum of the conductances attached to that node. Note also that the other nodal voltages within the same equation are multiplied by the negative of the conductance between the two nodes. The current sources are represented to the right of the equals sign with a positive sign if they supply current to the node and with a negative sign if they draw current from the node.

These conclusions can be expanded to include networks with any number of nodes. This will allow us to write nodal equations rapidly and in a form that is convenient for the use of determinants. A major requirement, however, is that *all voltage sources must first be converted to current sources before the procedure is applied.* Note the parallelism between the following four steps of application and those required for mesh analysis in Section 8.8:

1. ***Choose a reference node and assign a subscripted voltage label to the (N − 1) remaining nodes of the network.***
2. ***The number of equations required for a complete solution is equal to the number of subscripted voltages (N − 1). Column 1 of each equation is formed by summing the conductances tied to the node of interest and multiplying the result by that subscripted nodal voltage.***
3. ***We must now consider the mutual terms that, as noted in the preceding example, are always subtracted from the first column. It is possible to have more than one mutual term if the nodal voltage of current interest has an element in common with more than one other nodal voltage. This will be demonstrated in an example to follow. Each mutual term is the product of the mutual conductance and the other nodal voltage tied to that conductance.***
4. ***The column to the right of the equality sign is the algebraic sum of the current sources tied to the node of interest. A current source is assigned a positive sign if it supplies current to a node and a negative sign if it draws current from the node.***
5. ***Solve the resulting simultaneous equations for the desired voltages.***

Let us now consider a few examples.

EXAMPLE 8.23 Write the nodal equations for the network of Fig. 8.53.

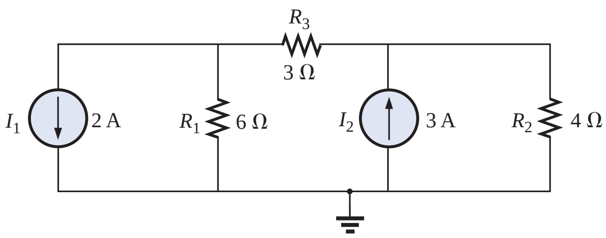


FIG. 8.53
Example 8.23.

Solution:

Step 1: The figure is redrawn with assigned subscripted voltages in Fig. 8.54.

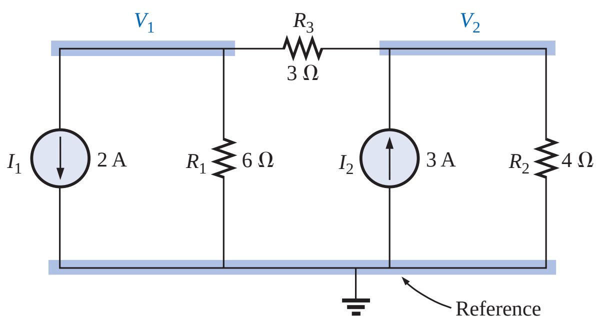


FIG. 8.54
Defining the nodes for the network of Fig. 8.53.

Steps 2 to 4:

$$V_1\!: \quad \underbrace{\left(\frac{1}{6\ \Omega} + \frac{1}{3\ \Omega}\right)}_{\substack{\text{Sum of}\\ \text{conductances}\\ \text{connected}\\ \text{to node 1}}} V_1 - \underbrace{\left(\frac{1}{3\ \Omega}\right)}_{\substack{\text{Mutual}\\ \text{conductance}}} V_2 = \overset{\substack{\text{Drawing current}\\ \text{from node 1}\\ \downarrow}}{-2\ \text{A}}$$

$$V_2\!: \quad \underbrace{\left(\frac{1}{4\ \Omega} + \frac{1}{3\ \Omega}\right)}_{\substack{\text{Sum of}\\ \text{conductances}\\ \text{connected}\\ \text{to node 2}}} V_2 - \underbrace{\left(\frac{1}{3\ \Omega}\right)}_{\substack{\text{Mutual}\\ \text{conductance}}} V_1 = \overset{\substack{\text{Supplying current}\\ \text{to node 2}\\ \downarrow}}{+3\ \text{A}}$$

and

$$\frac{1}{2}V_1 - \frac{1}{3}V_2 = -2$$

$$-\frac{1}{3}V_1 + \frac{7}{12}V_2 = 3$$

EXAMPLE 8.24 Find the voltage across the 3-Ω resistor of Fig. 8.55 by nodal analysis.

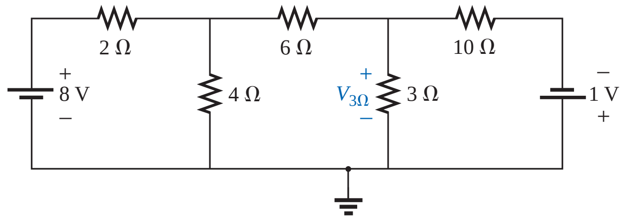


FIG. 8.55
Example 8.24.

Solution: Converting sources and choosing nodes (Fig. 8.56), we have

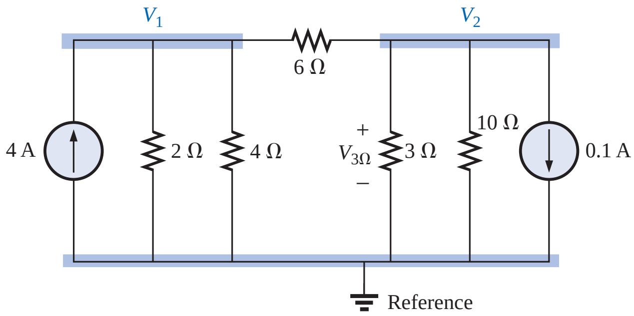


FIG. 8.56
Defining the nodes for the network of Fig. 8.55.

$$\left.\begin{array}{l} \left(\frac{1}{2\ \Omega}+\frac{1}{4\ \Omega}+\frac{1}{6\ \Omega}\right)V_1-\left(\frac{1}{6\ \Omega}\right)V_2=+4\text{ A} \\ \left(\frac{1}{10\ \Omega}+\frac{1}{3\ \Omega}+\frac{1}{6\ \Omega}\right)V_2-\left(\frac{1}{6\ \Omega}\right)V_1=-0.1\text{ A} \end{array}\right\}$$

$$\frac{11}{12}V_1-\frac{1}{6}V_2=4$$
$$-\frac{1}{6}V_1+\frac{3}{5}V_2=-0.1$$

resulting in

$$\begin{aligned} 11V_1-\ \ 2V_2&=+48 \\ -5V_1+18V_2&=-3 \end{aligned}$$

and

$$V_2=V_{3\Omega}=\frac{\begin{vmatrix}11 & 48\\ -5 & -3\end{vmatrix}}{\begin{vmatrix}11 & -2\\ -5 & 18\end{vmatrix}}=\frac{-33+240}{198-10}=\frac{207}{188}=\mathbf{1.101\ V}$$

As demonstrated for mesh analysis, nodal analysis can also be a very useful technique for solving networks with only one source.

EXAMPLE 8.25 Using nodal analysis, determine the potential across the 4-Ω resistor in Fig. 8.57.

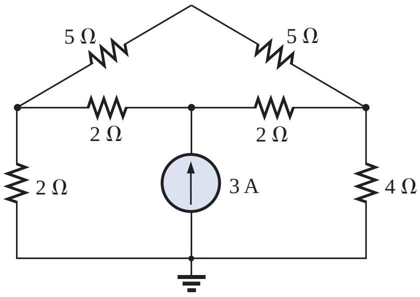


FIG. 8.57
Example 8.25.

Solution 1: The reference and four subscripted voltage levels were chosen as shown in Fig. 8.58. A moment of reflection should reveal that for any difference in potential between V_1 and V_3, the current through and the potential drop across each 5-Ω resistor will be the same. Therefore, V_4 is simply a midvoltage level between V_1 and V_3 and is known if V_1 and V_3 are available. We will therefore not include it in a nodal voltage and will redraw the network as shown in Fig. 8.59. Understand, however, that V_4 can be included if desired, although four nodal voltages will result rather than the three to be obtained in the solution of this problem.

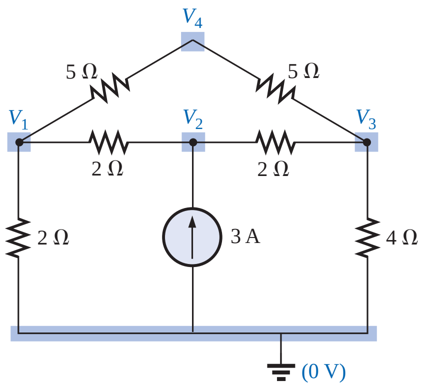


FIG. 8.58
Defining the nodes for the network of Fig. 8.57.

$$
\begin{aligned}
V_1&: \quad \left(\frac{1}{2\ \Omega}+\frac{1}{2\ \Omega}+\frac{1}{10\ \Omega}\right)V_1-\left(\frac{1}{2\ \Omega}\right)V_2-\left(\frac{1}{10\ \Omega}\right)V_3=0\\
V_2&: \quad \left(\frac{1}{2\ \Omega}+\frac{1}{2\ \Omega}\right)V_2-\left(\frac{1}{2\ \Omega}\right)V_1-\left(\frac{1}{2\ \Omega}\right)V_3=3\text{ A}\\
V_3&: \quad \left(\frac{1}{10\ \Omega}+\frac{1}{2\ \Omega}+\frac{1}{4\ \Omega}\right)V_3-\left(\frac{1}{2\ \Omega}\right)V_2-\left(\frac{1}{10\ \Omega}\right)V_1=0
\end{aligned}
$$

which are rewritten as

$$
\begin{aligned}
1.1V_1-0.5V_2-0.1V_3&=0\\
V_2-0.5V_1-0.5V_3&=3\\
0.85V_3-0.5V_2-0.1V_1&=0
\end{aligned}
$$

For determinants,

$$
\begin{aligned}
1.1V_1-0.5V_2-\ 0.1V_3&=0\\
-0.5V_1+\ \ 1V_2-\ 0.5V_3&=3\\
-0.1V_1-0.5V_2+0.85V_3&=0
\end{aligned}
$$

Before continuing, note the symmetry about the major diagonal in the equation above. Recall a similar result for mesh analysis. Examples 8.23 and 8.24 also exhibit this property in the resulting equations. Keep this thought in mind as a check on future applications of nodal analysis.

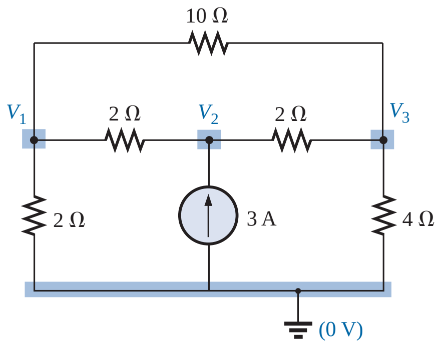


FIG. 8.59
Reducing the number of nodes for the network of Fig. 8.57 by combining the two 5-Ω resistors.

$$
V_3=V_{4\Omega}=\frac{\begin{vmatrix}1.1 & -0.5 & 0\\ -0.5 & +1 & 3\\ -0.1 & -0.5 & 0\end{vmatrix}}{\begin{vmatrix}1.1 & -0.5 & -0.1\\ -0.5 & +1 & -0.5\\ -0.1 & -0.5 & +0.85\end{vmatrix}}=\mathbf{4.645\ V}
$$

Mathcad Solution: By now the sequence of steps necessary to solve a series of equations using Mathcad should be quite familiar and less threatening than the first encounter. For this example, all the parameters were entered in the three simultaneous equations, avoiding the need to define each parameter of the network. Simply provide a **Guess** at the three nodal voltages, apply the word **Given,** and enter the three equations properly as shown in Fig. 8.60. It does take some practice to ensure that the bracket is moved to the proper location before making an entry, but this is simply part of the rules set up to maintain control of the operations to be performed. Finally, request the desired nodal voltages using the correct format. The numerical results will appear, again confirming our theoretical solutions.

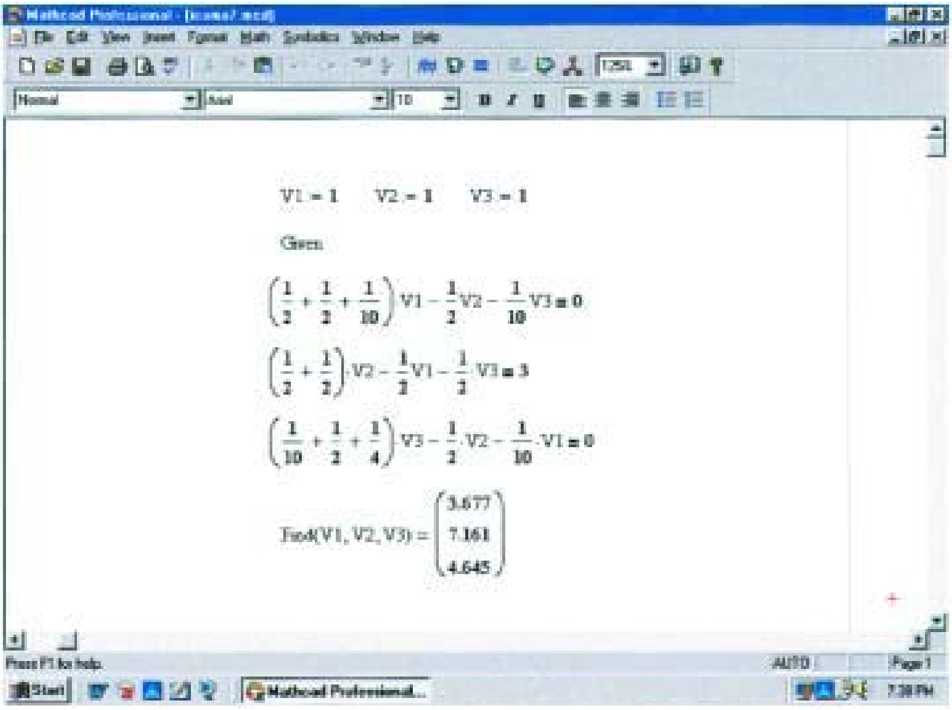


FIG. 8.60
Using Mathcad to verify the mathematical calculations of Example 8.25.

The next example has only one source applied to a ladder network.

EXAMPLE 8.26 Write the nodal equations and find the voltage across the 2-Ω resistor for the network of Fig. 8.61.

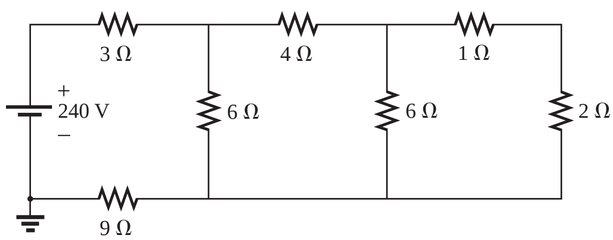


FIG. 8.61
Example 8.26.

Solution: The nodal voltages are chosen as shown in Fig. 8.62.

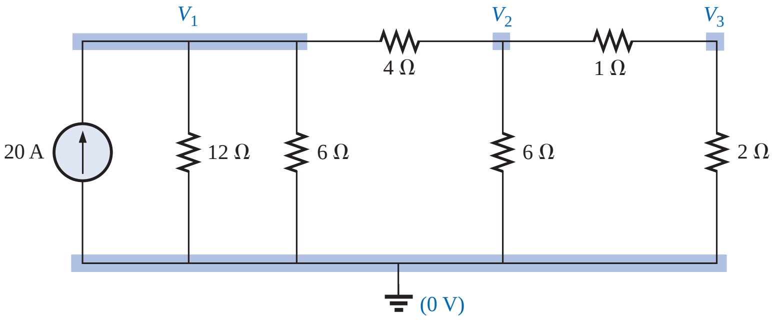


FIG. 8.62
Converting the voltage source to a current source and defining the nodes for the network of Fig. 8.61.

$$
\begin{aligned}
V_1&: \quad \left(\frac{1}{12\,\Omega}+\frac{1}{6\,\Omega}+\frac{1}{4\,\Omega}\right)V_1-\left(\frac{1}{4\,\Omega}\right)V_2+\quad 0 \quad = 20\text{ V}\\
V_2&: \quad \left(\frac{1}{4\,\Omega}+\frac{1}{6\,\Omega}+\frac{1}{1\,\Omega}\right)V_2-\left(\frac{1}{4\,\Omega}\right)V_1-\left(\frac{1}{1\,\Omega}\right)V_3 = 0\\
V_3&: \quad \left(\frac{1}{1\,\Omega}+\frac{1}{2\,\Omega}\right)V_3-\left(\frac{1}{1\,\Omega}\right)V_2+\quad 0 \quad = 0
\end{aligned}
$$

and

$$
\begin{aligned}
0.5V_1 - 0.25V_2 + 0 \quad &= 20\\
-0.25V_1 + \frac{17}{12}V_2 \quad - 1V_3 \quad &= 0\\
0 \; - 1V_2 \quad + 1.5V_3 &= 0
\end{aligned}
$$

Note the symmetry present about the major axis. Application of determinants reveals that

$$V_3 = V_{2\Omega} = \mathbf{10.667\text{ V}}$$

8.11 BRIDGE NETWORKS

This section introduces the **bridge network,** a configuration that has a multitude of applications. In the chapters to follow, it will be employed in both dc and ac meters. In the electronics courses it will be encountered early in the discussion of rectifying circuits employed in converting a varying signal to one of a steady nature (such as dc). A number of other areas of application also require some knowledge of ac networks; these areas will be discussed later.

The bridge network may appear in one of the three forms as indicated in Fig. 8.63. The network of Fig. 8.63(c) is also called a *symmetrical lattice network* if $R_2 = R_3$ and $R_1 = R_4$. Figure 8.63(c) is an excellent example of how a planar network can be made to appear nonplanar. For the purposes of investigation, let us examine the network of Fig. 8.64 using mesh and nodal analysis.

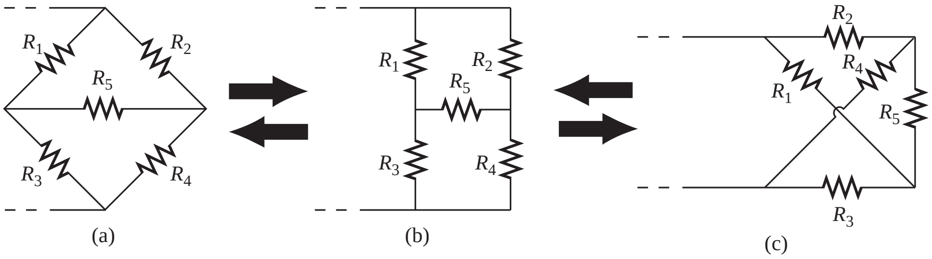


FIG. 8.63
Various formats for a bridge network.

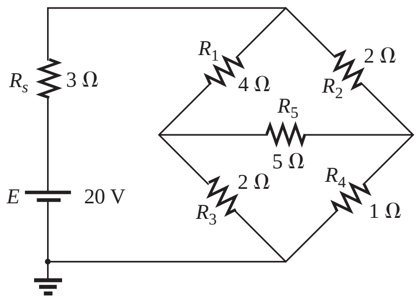


FIG. 8.64
Standard bridge configuration.

Mesh analysis (Fig. 8.65) yields

$$(3\ \Omega + 4\ \Omega + 2\ \Omega)I_1 - (4\ \Omega)I_2 - (2\ \Omega)I_3 = 20\text{ V}$$
$$(4\ \Omega + 5\ \Omega + 2\ \Omega)I_2 - (4\ \Omega)I_1 - (5\ \Omega)I_3 = 0$$
$$(2\ \Omega + 5\ \Omega + 1\ \Omega)I_3 - (2\ \Omega)I_1 - (5\ \Omega)I_2 = 0$$

and

$$9I_1 - 4I_2 - 2I_3 = 20$$
$$-4I_1 + 11I_2 - 5I_3 = 0$$
$$-2I_1 - 5I_2 + 8I_3 = 0$$

with the result that

$$I_1 = \mathbf{4\ A}$$
$$I_2 = \mathbf{2.667\ A}$$
$$I_3 = \mathbf{2.667\ A}$$

The net current through the 5-Ω resistor is

$$I_{5\Omega} = I_2 - I_3 = \mathbf{2.667\ A - 2.667\ A = 0\ A}$$

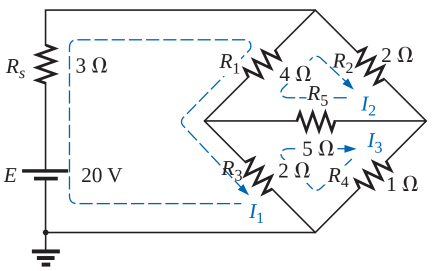


FIG. 8.65
Assigning the mesh currents to the network of Fig. 8.64.

Nodal analysis (Fig. 8.66) yields

$$\left(\frac{1}{3\ \Omega} + \frac{1}{4\ \Omega} + \frac{1}{2\ \Omega}\right)V_1 - \left(\frac{1}{4\ \Omega}\right)V_2 - \left(\frac{1}{2\ \Omega}\right)V_3 = \frac{20}{3}\text{ A}$$
$$\left(\frac{1}{4\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{5\ \Omega}\right)V_2 - \left(\frac{1}{4\ \Omega}\right)V_1 - \left(\frac{1}{5\ \Omega}\right)V_3 = 0$$
$$\left(\frac{1}{5\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{1\ \Omega}\right)V_3 - \left(\frac{1}{2\ \Omega}\right)V_1 - \left(\frac{1}{5\ \Omega}\right)V_2 = 0$$

and

$$\left(\frac{1}{3\ \Omega} + \frac{1}{4\ \Omega} + \frac{1}{2\ \Omega}\right)V_1 - \left(\frac{1}{4\ \Omega}\right)V_2 - \left(\frac{1}{2\ \Omega}\right)V_3 = \frac{20}{3}\text{ A}$$
$$-\left(\frac{1}{4\ \Omega}\right)V_1 + \left(\frac{1}{4\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{5\ \Omega}\right)V_2 - \left(\frac{1}{5\ \Omega}\right)V_3 = 0$$
$$-\left(\frac{1}{2\ \Omega}\right)V_1 - \left(\frac{1}{5\ \Omega}\right)V_2 + \left(\frac{1}{5\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{1\ \Omega}\right)V_3 = 0$$

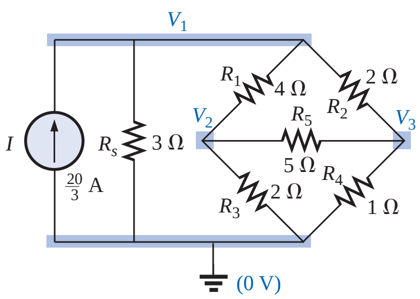


FIG. 8.66
Defining the nodal voltages for the network of Fig. 8.64.

Note the symmetry of the solution.

With the TI-86 calculator, the top part of the determinant is determined by the following (take note of the calculations within parentheses):

det[[20/3,−1/4,−1/2][0,(1/4+1/2+1/5),−1/5][0,−1/5,(1/5+1/2+1/1)]] ENTER	10.5

CALC. 8.4

with the bottom of the determinant determined by:

det[[(1/3+1/4+1/2),−1/4,−1/2][−1/4,(1/4+1/2+1/5),−1/5][−1/2,−1/5,(1/5+1/2+1/1)]] ENTER	1.312

CALC. 8.5

Finally,

10.5/1.312 ENTER	8

CALC. 8.6

and $V_1 = \mathbf{8\ V}$

Similarly, $V_2 = \mathbf{2.667\ V}$ and $V_3 = \mathbf{2.667\ V}$

and the voltage across the 5-Ω resistor is

$$V_{5\Omega} = V_2 - V_3 = \mathbf{2.667\ V - 2.667\ V = 0\ V}$$

Since $V_{5\Omega} = 0$ V, we can insert a short in place of the bridge arm without affecting the network behavior. (Certainly $V = IR = I\cdot(0) = 0$ V.) In Fig. 8.67, a short circuit has replaced the resistor R_5, and the voltage across R_4 is to be determined. The network is redrawn in Fig. 8.68, and

$$V_{1\Omega} = \frac{(2\ \Omega \parallel 1\ \Omega)20\ \text{V}}{(2\ \Omega \parallel 1\ \Omega) + (4\ \Omega \parallel 2\ \Omega) + 3\ \Omega} \quad \text{(voltage divider rule)}$$

$$= \frac{\frac{2}{3}(20\ \text{V})}{\frac{2}{3} + \frac{8}{6} + 3} = \frac{\frac{2}{3}(20\ \text{V})}{\frac{2}{3} + \frac{4}{3} + \frac{9}{3}}$$

$$= \frac{2(20\ \text{V})}{2 + 4 + 9} = \frac{40\ \text{V}}{15} = \mathbf{2.667\ V}$$

as obtained earlier.

We found through mesh analysis that $I_{5\Omega} = 0$ A, which has as its equivalent an open circuit as shown in Fig. 8.69(a). (Certainly $I = V/R = 0/(\infty\ \Omega) = 0$ A.) The voltage across the resistor R_4 will again be determined and compared with the result above.

The network is redrawn after combining series elements, as shown in Fig. 8.69(b), and

$$V_{3\Omega} = \frac{(6\ \Omega \parallel 3\ \Omega)(20\ \text{V})}{6\ \Omega \parallel 3\ \Omega + 3\ \Omega} = \frac{2\ \Omega(20\ \text{V})}{2\ \Omega + 3\ \Omega} = 8\ \text{V}$$

and
$$V_{1\Omega} = \frac{1\ \Omega(8\ \text{V})}{1\ \Omega + 2\ \Omega} = \frac{8\ \text{V}}{3} = \mathbf{2.667\ V}$$

as above.

The condition $V_{5\Omega} = 0$ V or $I_{5\Omega} = 0$ A exists only for a particular relationship between the resistors of the network. Let us now derive this relationship using the network of Fig. 8.70, in which it is indicated that $I = 0$ A and $V = 0$ V. Note that resistor R_s of the network of Fig. 8.69 will not appear in the following analysis.

The bridge network is said to be *balanced* when the condition of $I = 0$ A or $V = 0$ V exists.

If $V = 0$ V (short circuit between a and b), then

$$V_1 = V_2$$

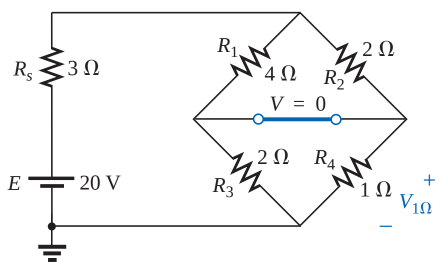


FIG. 8.67
Substituting the short-circuit equivalent for the balance arm of a balanced bridge.

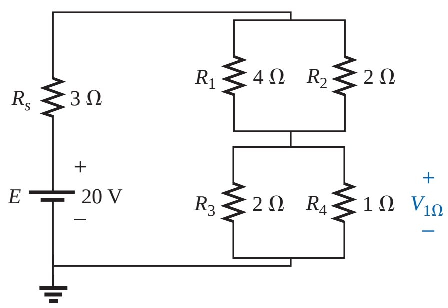
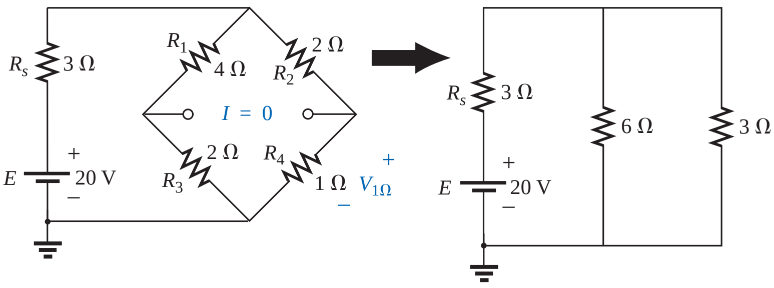


FIG. 8.68
Redrawing the network of Fig. 8.67.



(a) (b)

FIG. 8.69
Substituting the open-circuit equivalent for the balance arm of a balanced bridge.

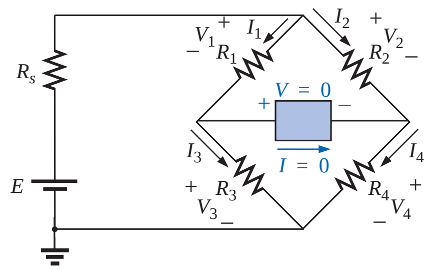


FIG. 8.70
Establishing the balance criteria for a bridge network.

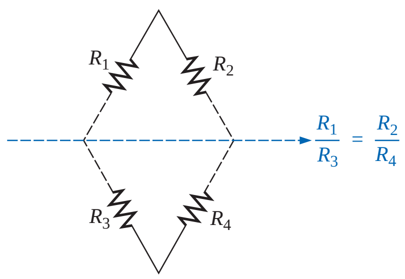


FIG. 8.71
A visual approach to remembering the balance condition.

and
$$I_1R_1 = I_2R_2$$
or
$$I_1 = \frac{I_2R_2}{R_1}$$
In addition, when $V = 0$ V,
$$V_3 = V_4$$
and
$$I_3R_3 = I_4R_4$$

If we set $I = 0$ A, then $I_3 = I_1$ and $I_4 = I_2$, with the result that the above equation becomes
$$I_1R_3 = I_2R_4$$
Substituting for I_1 from above yields
$$\left(\frac{I_2R_2}{R_1}\right)R_3 = I_2R_4$$
or, rearranging, we have
$$\boxed{\frac{R_1}{R_3} = \frac{R_2}{R_4}} \tag{8.4}$$

This conclusion states that if the ratio of R_1 to R_3 is equal to that of R_2 to R_4, the bridge will be balanced, and $I = 0$ A or $V = 0$ V. A method of memorizing this form is indicated in Fig. 8.71.

For the example above, $R_1 = 4\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 2\ \Omega$, $R_4 = 1\ \Omega$, and
$$\frac{R_1}{R_3} = \frac{R_2}{R_4} \rightarrow \frac{4\ \Omega}{2\ \Omega} = \frac{2\ \Omega}{1\ \Omega} = 2$$

The emphasis in this section has been on the balanced situation. Understand that if the ratio is not satisfied, there will be a potential drop across the balance arm and a current through it. The methods just described (mesh and nodal analysis) will yield any and all potentials or currents desired, just as they did for the balanced situation.

8.12 Y-Δ (T-π) AND Δ-Y (π-T) CONVERSIONS

Circuit configurations are often encountered in which the resistors do not appear to be in series or parallel. Under these conditions, it may be necessary to convert the circuit from one form to another to solve for any unknown quantities if mesh or nodal analysis is not applied. Two circuit configurations that often account for these difficulties are the **wye (Y)** and **delta (Δ) configurations,** depicted in Fig. 8.72(a). They are also referred to as the **tee (T)** and **pi (π),** respectively, as indicated in Fig. 8.72(b). Note that the pi is actually an inverted delta.

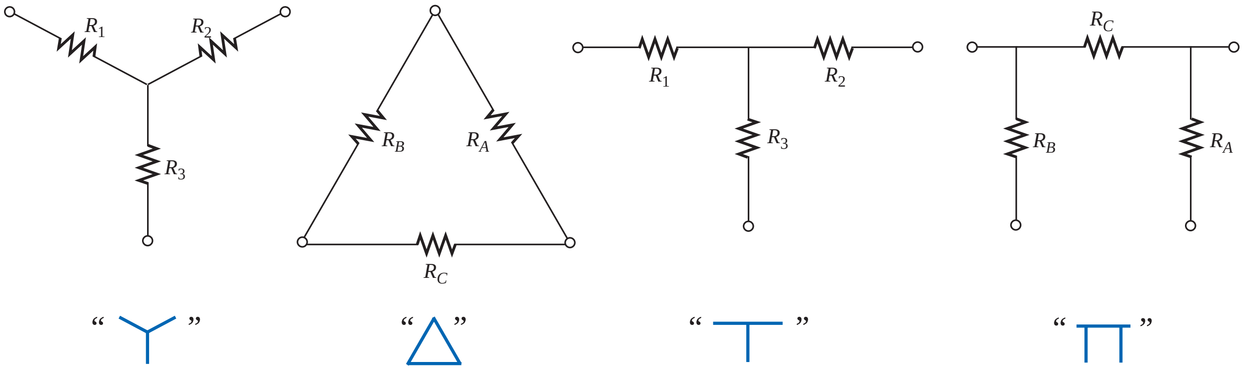


FIG. 8.72
The Y (T) and Δ (π) configurations.

The purpose of this section is to develop the equations for converting from Δ to Y, or vice versa. This type of conversion will normally lead to a network that can be solved using techniques such as those described in Chapter 7. In other words, in Fig. 8.73, with terminals *a*, *b*, and *c* held fast, if the wye (Y) configuration were desired *instead of* the inverted delta (Δ) configuration, all that would be necessary is a direct application of the equations to be derived. The phrase *instead of* is emphasized to ensure that it is understood that only one of these configurations is to appear at one time between the indicated terminals.

It is our purpose (referring to Fig. 8.73) to find some expression for R_1, R_2, and R_3 in terms of R_A, R_B, and R_C, and vice versa, that will ensure that the resistance between any two terminals of the Y configuration will be the same with the Δ configuration inserted in place of the Y configuration (and vice versa). If the two circuits are to be equivalent, the total resistance between any two terminals must be the same. Consider terminals *a*-*c* in the Δ-Y configurations of Fig. 8.74.

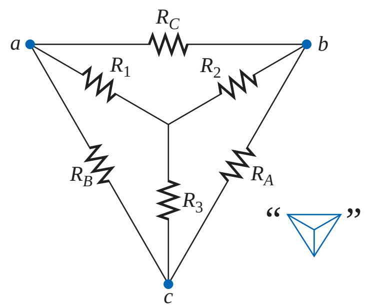


FIG. 8.73
Introducing the concept of Δ-Y or Y-Δ conversions.

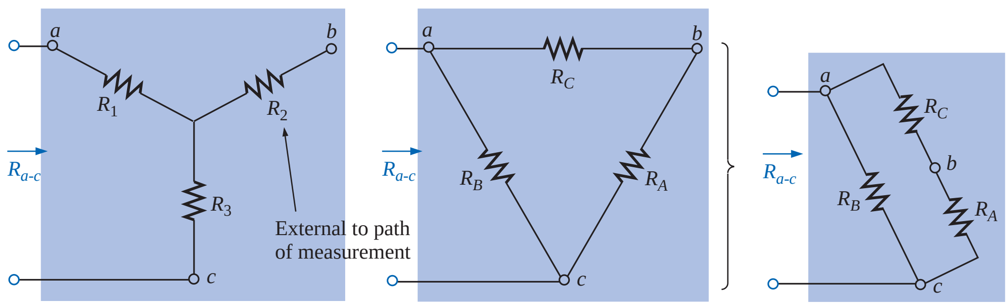


FIG. 8.74
Finding the resistance $R_{a\text{-}c}$ for the Y and Δ configurations.

Let us first assume that we want to convert the Δ (R_A, R_B, R_C) to the Y (R_1, R_2, R_3). This requires that we have a relationship for R_1, R_2, and R_3 in terms of R_A, R_B, and R_C. If the resistance is to be the same between terminals *a-c* for both the Δ and the Y, the following must be true:

$$R_{a\text{-}c}\,(\text{Y}) = R_{a\text{-}c}\,(\Delta)$$

so that

$$\boxed{R_{a\text{-}c} = R_1 + R_3 = \frac{R_B(R_A + R_C)}{R_B + (R_A + R_C)}} \qquad \textbf{(8.5a)}$$

Using the same approach for *a-b* and *b-c*, we obtain the following relationships:

$$\boxed{R_{a\text{-}b} = R_1 + R_2 = \frac{R_C(R_A + R_B)}{R_C + (R_A + R_B)}} \qquad \textbf{(8.5b)}$$

and

$$\boxed{R_{b\text{-}c} = R_2 + R_3 = \frac{R_A(R_B + R_C)}{R_A + (R_B + R_C)}} \qquad \textbf{(8.5c)}$$

Subtracting Eq. (8.5a) from Eq. (8.5b), we have

$$(R_1 + R_2) - (R_1 + R_3) = \left(\frac{R_C R_B + R_C R_A}{R_A + R_B + R_C}\right) - \left(\frac{R_B R_A + R_B R_C}{R_A + R_B + R_C}\right)$$

so that

$$\boxed{R_2 - R_3 = \frac{R_A R_C - R_B R_A}{R_A + R_B + R_C}} \qquad \textbf{(8.5d)}$$

Subtracting Eq. (8.5d) from Eq. (8.5c) yields

$$(R_2 + R_3) - (R_2 - R_3) = \left(\frac{R_A R_B + R_A R_C}{R_A + R_B + R_C}\right) - \left(\frac{R_A R_C - R_B R_A}{R_A + R_B + R_C}\right)$$

so that

$$2R_3 = \frac{2R_B R_A}{R_A + R_B + R_C}$$

resulting in the following expression for R_3 in terms of R_A, R_B, and R_C:

$$\boxed{R_3 = \frac{R_A R_B}{R_A + R_B + R_C}} \qquad \textbf{(8.6a)}$$

Following the same procedure for R_1 and R_2, we have

$$\boxed{R_1 = \frac{R_B R_C}{R_A + R_B + R_C}} \qquad \textbf{(8.6b)}$$

and

$$\boxed{R_2 = \frac{R_A R_C}{R_A + R_B + R_C}} \qquad \textbf{(8.6c)}$$

Note that each resistor of the Y is equal to the product of the resistors in the two closest branches of the Δ divided by the sum of the resistors in the Δ.

To obtain the relationships necessary to convert from a Y to a Δ, first divide Eq. (8.6a) by Eq. (8.6b):

$$\frac{R_3}{R_1} = \frac{(R_A R_B)/(R_A + R_B + R_C)}{(R_B R_C)/(R_A + R_B + R_C)} = \frac{R_A}{R_C}$$

or

$$R_A = \frac{R_C R_3}{R_1}$$

Then divide Eq. (8.6a) by Eq. (8.6c):

$$\frac{R_3}{R_2} = \frac{(R_A R_B)/(R_A + R_B + R_C)}{(R_A R_C)/(R_A + R_B + R_C)} = \frac{R_B}{R_C}$$

or

$$R_B = \frac{R_3 R_C}{R_2}$$

Substituting for R_A and R_B in Eq. (8.6c) yields

$$R_2 = \frac{(R_C R_3/R_1)R_C}{(R_3 R_C/R_2) + (R_C R_3/R_1) + R_C}$$

$$= \frac{(R_3/R_1)R_C}{(R_3/R_2) + (R_3/R_1) + 1}$$

Placing these over a common denominator, we obtain

$$R_2 = \frac{(R_3 R_C/R_1)}{(R_1 R_2 + R_1 R_3 + R_2 R_3)/(R_1 R_2)}$$

$$= \frac{R_2 R_3 R_C}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

and

$$\boxed{R_C = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}} \tag{8.7a}$$

We follow the same procedure for R_B and R_A:

$$\boxed{R_A = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}} \tag{8.7b}$$

and

$$\boxed{R_B = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}} \tag{8.7c}$$

Note that the value of each resistor of the Δ is equal to the sum of the possible product combinations of the resistances of the Y divided by the resistance of the Y farthest from the resistor to be determined.

Let us consider what would occur if all the values of a Δ or Y were the same. If $R_A = R_B = R_C$, Equation (8.6a) would become (using R_A only) the following:

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{R_A R_A}{R_A + R_A + R_A} = \frac{R_A^2}{3R_A} = \frac{R_A}{3}$$

and, following the same procedure,

$$R_1 = \frac{R_A}{3} \qquad R_2 = \frac{R_A}{3}$$

In general, therefore,

$$\boxed{R_Y = \frac{R_\Delta}{3}} \qquad \textbf{(8.8a)}$$

or

$$\boxed{R_\Delta = 3R_Y} \qquad \textbf{(8.8b)}$$

which indicates that *for a Y of three equal resistors, the value of each resistor of the Δ is equal to three times the value of any resistor of the Y*. If only two elements of a Y or a Δ are the same, the corresponding Δ or Y of each will also have two equal elements. The converting of equations will be left as an exercise for the reader.

The Y and the Δ will often appear as shown in Fig. 8.75. They are then referred to as a **tee (T)** and a **pi (π)** network, respectively. The equations used to convert from one form to the other are exactly the same as those developed for the Y and Δ transformation.

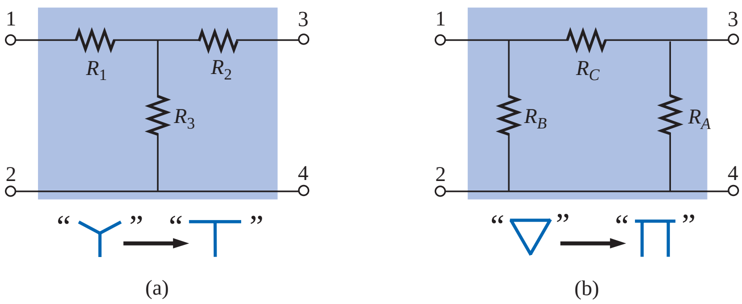


FIG. 8.75
The relationship between the Y and T configurations and the Δ and π configurations.

EXAMPLE 8.27 Convert the Δ of Fig. 8.76 to a Y.

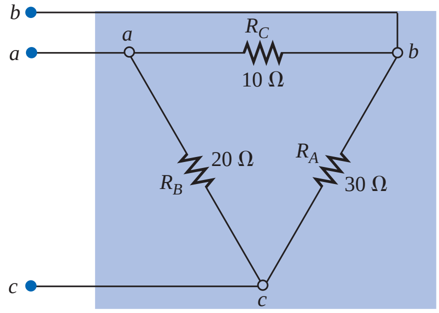


FIG. 8.76
Example 8.27.

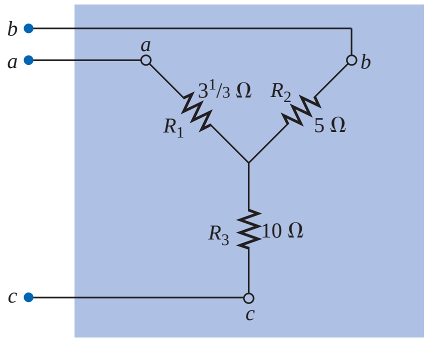


FIG. 8.77
The Y equivalent for the Δ of Fig. 8.76.

Solution:

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(20\ \Omega)(10\ \Omega)}{30\ \Omega + 20\ \Omega + 10\ \Omega} = \frac{200\ \Omega}{60} = \mathbf{3\tfrac{1}{3}\ \Omega}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(30\ \Omega)(10\ \Omega)}{60\ \Omega} = \frac{300\ \Omega}{60} = \mathbf{5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(20\ \Omega)(30\ \Omega)}{60\ \Omega} = \frac{600\ \Omega}{60} = \mathbf{10\ \Omega}$$

The equivalent network is shown in Fig. 8.77 (page 298).

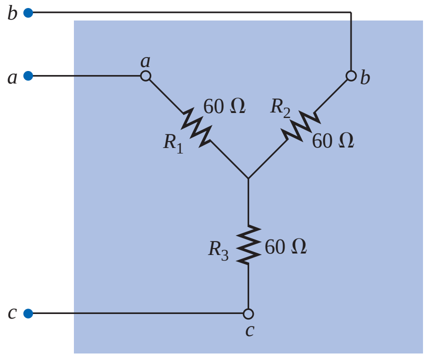


FIG. 8.78
Example 8.28.

EXAMPLE 8.28 Convert the Y of Fig. 8.78 to a **Δ.**

Solution:

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$
$$= \frac{(60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega)}{60\ \Omega}$$
$$= \frac{3600\ \Omega + 3600\ \Omega + 3600\ \Omega}{60} = \frac{10{,}800\ \Omega}{60}$$
$$R_A = \mathbf{180\ \Omega}$$

However, the three resistors for the Y are equal, permitting the use of Eq. (8.8) and yielding

$$R_\Delta = 3R_Y = 3(60\ \Omega) = 180\ \Omega$$

and
$$R_B = R_C = \mathbf{180\ \Omega}$$

The equivalent network is shown in Fig. 8.79.

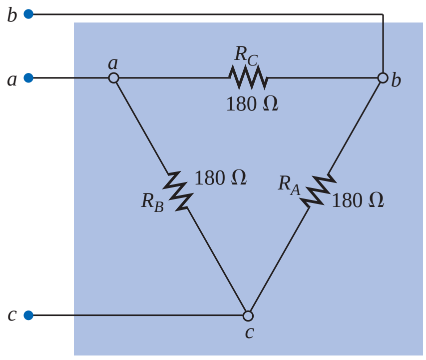


FIG. 8.79
The Δ equivalent for the Y of Fig. 8.78.

EXAMPLE 8.29 Find the total resistance of the network of Fig. 8.80, where $R_A = 3\ \Omega$, $R_B = 3\ \Omega$, and $R_C = 6\ \Omega$.

Solution:

Two resistors of the Δ were equal; therefore, two resistors of the Y will be equal.

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{12\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(3\ \Omega)(3\ \Omega)}{12\ \Omega} = \frac{9\ \Omega}{12} = \mathbf{0.75\ \Omega}$$

Replacing the Δ by the Y, as shown in Fig. 8.81, yields

$$R_T = 0.75\ \Omega + \frac{(4\ \Omega + 1.5\ \Omega)(2\ \Omega + 1.5\ \Omega)}{(4\ \Omega + 1.5\ \Omega) + (2\ \Omega + 1.5\ \Omega)}$$
$$= 0.75\ \Omega + \frac{(5.5\ \Omega)(3.5\ \Omega)}{5.5\ \Omega + 3.5\ \Omega}$$
$$= 0.75\ \Omega + 2.139\ \Omega$$
$$R_T = \mathbf{2.889\ \Omega}$$

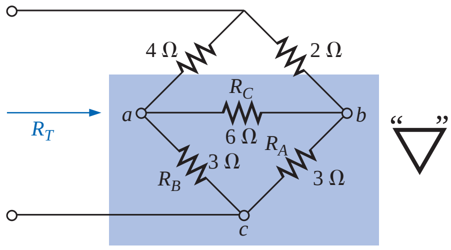


FIG. 8.80
Example 8.29.

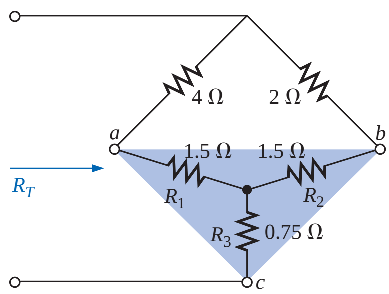


FIG. 8.81
Substituting the Y equivalent for the bottom Δ of Fig. 8.80.

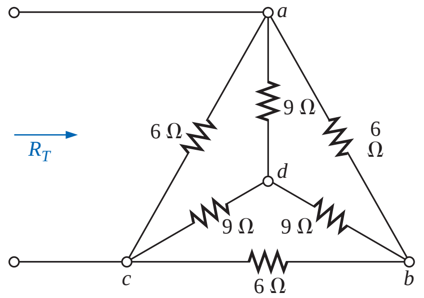


FIG. 8.82
Example 8.30.

EXAMPLE 8.30 Find the total resistance of the network of Fig. 8.82.

Solutions: Since all the resistors of the Δ or Y are the same, Equations (8.8a) and (8.8b) can be used to convert either form to the other.

a. *Converting the Δ to a Y.* Note: When this is done, the resulting d' of the new Y will be the same as the point d shown in the original figure, only because both systems are "balanced." That is, the resistance in each branch of each system has the same value:

$$R_Y = \frac{R_\Delta}{3} = \frac{6\ \Omega}{3} = 2\ \Omega \qquad \text{(Fig. 8.83)}$$

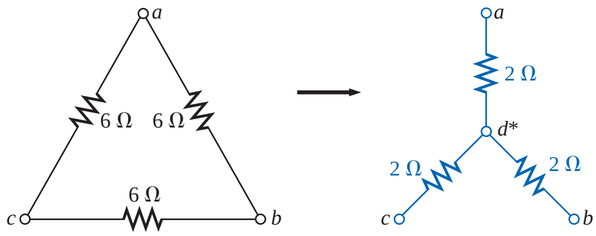


FIG. 8.83
Converting the Δ configuration of Fig. 8.82 to a Y configuration.

The network then appears as shown in Fig. 8.84.

$$R_T = 2\left[\frac{(2\ \Omega)(9\ \Omega)}{2\ \Omega + 9\ \Omega}\right] = \mathbf{3.2727\ \Omega}$$

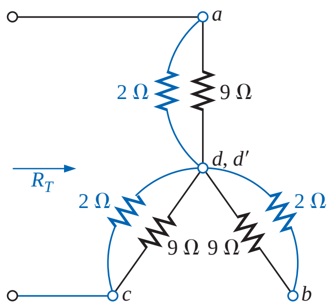


FIG. 8.84
Substituting the Y configuration for the converted Δ into the network of Fig. 8.82.

b. *Converting the Y to a Δ:*

$$R_\Delta = 3R_Y = (3)(9\ \Omega) = 27\ \Omega \qquad \text{(Fig. 8.85)}$$

$$R'_T = \frac{(6\ \Omega)(27\ \Omega)}{6\ \Omega + 27\ \Omega} = \frac{162\ \Omega}{33} = 4.9091\ \Omega$$

$$R_T = \frac{R'_T(R'_T + R'_T)}{R'_T + (R'_T + R'_T)} = \frac{R'_T 2R'_T}{3R'_T} = \frac{2R'_T}{3}$$

$$= \frac{2(4.9091\ \Omega)}{3} = \mathbf{3.2727\ \Omega}$$

which checks with the previous solution.

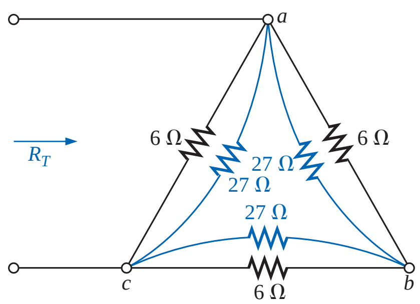


FIG. 8.85
Substituting the converted Y configuration into the network of Fig. 8.82.

8.13 APPLICATIONS

The Applications section of this chapter will discuss the constant current characteristic in the design of security systems, the bridge circuit in a common residential smoke detector, and the nodal voltages of a digital logic probe.

Constant Current Alarm Systems

The basic components of an alarm system employing a constant current supply are provided in Fig. 8.86. This design is improved over that provided in Chapter 5 in the sense that it is less sensitive to changes in resistance in the circuit due to heating, humidity, changes in the length of the line to the sensors, and so on. The 1.5-kΩ rheostat (total resistance between points *a* and *b*) is adjusted to ensure a current of 5 mA through the single-series security circuit. The adjustable rheostat is necessary to compensate for variations in the total resistance of the circuit introduced by the resistance of the wire, sensors, sensing relay, and milliammeter. The milliammeter is included to set the rheostat and ensure a current of 5 mA.

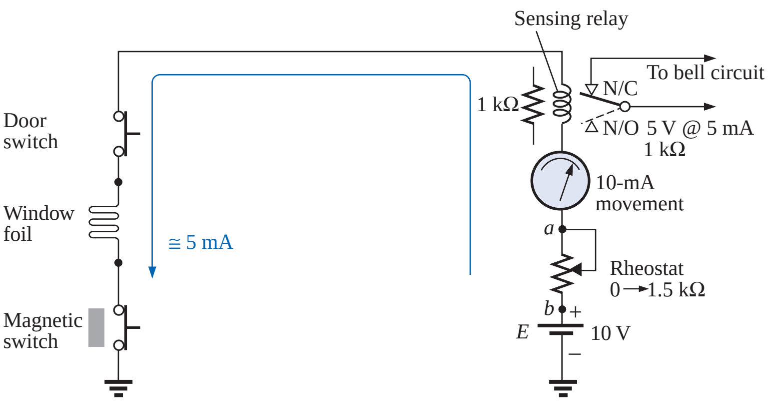


FIG. 8.86
Constant current alarm system.

If any of the sensors should open, the current through the entire circuit will drop to zero, the coil of the relay will release the plunger, and contact will be made with the N/C position of the relay. This action will complete the circuit for the bell circuit, and the alarm will sound. For the future, keep in mind that switch positions for a relay are always shown with no power to the network, resulting in the N/C position of Fig. 8.86. When power is applied, the switch will have the position indicated by the dashed line. That is, various factors, such as a change in resistance of any of the elements due to heating, humidity, and so on, would cause the applied voltage to redistribute itself and create a sensitive situation. With an adjusted 5 mA, the loading can change, but the current will always be 5 mA and the chance of tripping reduced. Take note of the fact that the relay is rated as 5 V at 5 mA, indicating that in the on state the voltage across the relay is 5 V and the current through the relay 5 mA. Its internal resistance is therefore 5 V/5 mA = 1 kΩ in this state.

A more advanced alarm system using a constant current is provided in Fig. 8.87. In this case an electronic system employing a single tran-

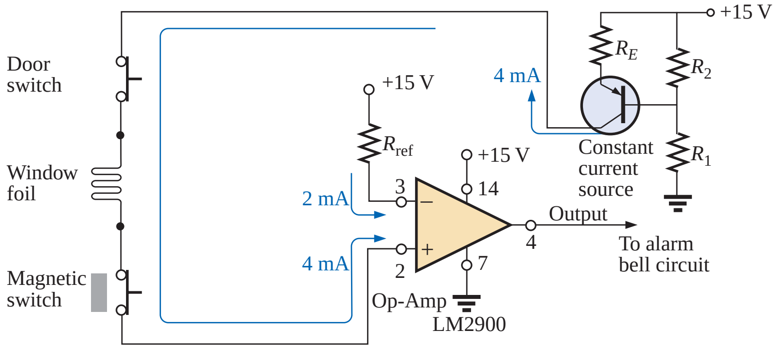


FIG. 8.87

Constant current alarm system with electronic components.

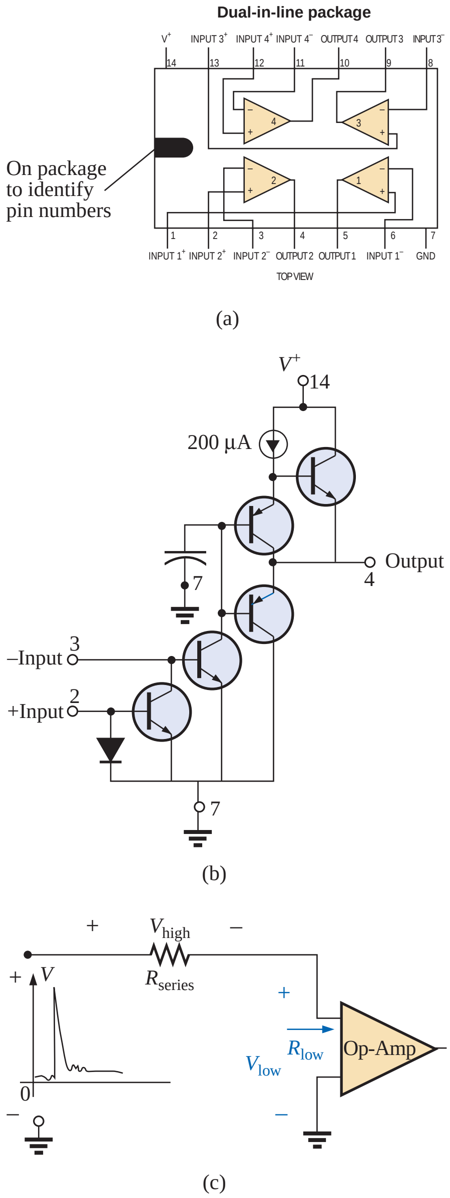


FIG. 8.88

LM2900 operational amplifier: (a) dual-in-line package (DIP); (b) components; (c) impact of low-input impedance.

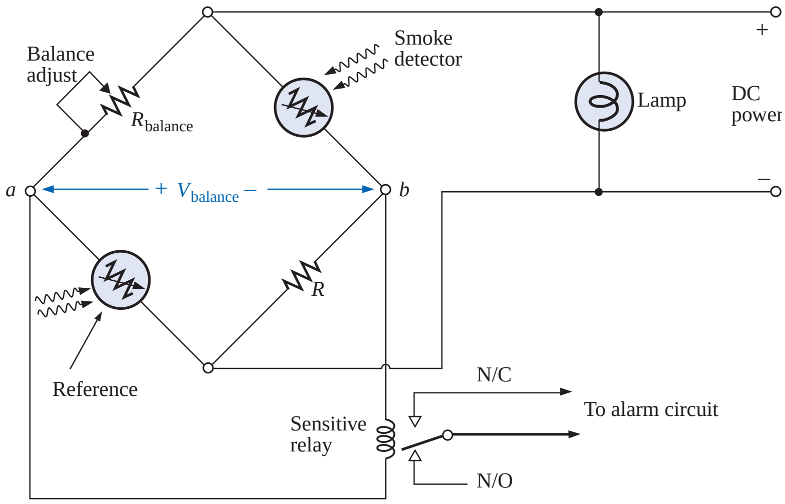
sistor, biasing resistors, and a dc battery are establishing a current of 4 mA through the series sensor circuit connected to the positive side of an operational amplifier (op-amp). Although the transistor and op-amp devices may be new to you, they will be discussed in detail in your electronics courses—you do not need to be aware of the details of their operation for now. Suffice it to say for the moment that the transistor in this application is being used not as an amplifier but as part of a design to establish a constant current through the circuit. The op-amp is a very useful component of numerous electronic systems, and it has important terminal characteristics established by a variety of components internal to its design. The LM2900 operational amplifier of Fig. 8.87 is one of four found in the dual-in-line integrated circuit package appearing in Fig. 8.88(a). Pins 2, 3, 4, 7, and 14 were used for the design of Fig. 8.87. Note in Fig. 8.88(b) the number of elements required to establish the desired terminal characteristics—the details of which will be investigated in your electronics courses.

In Fig. 8.87, the designed 15-V dc supply, biasing resistors, and transistor in the upper right-hand corner of the schematic establish a constant 4-mA current through the circuit. It is referred to as a *constant current source* because the current will remain fairly constant at 4 mA even though there may be moderate variations in the total resistance of the series sensor circuit connected to the transistor. Following the 4 mA through the circuit, we find that it enters terminal 2 (positive side of the input) of the op-amp. A second current of 2 mA, called the *reference current*, is established by the 15-V source and resistance R and enters terminal 3 (negative side of the input) of the op-amp. The reference current of 2 mA is necessary to establish a current for the 4-mA current of the network to be compared against. So long as the 4-mA current exists, the operational amplifier will provide a “high” output voltage that exceeds 13.5 V, with a typical level of 14.2 V (according to the specification sheet for the op-amp). However, if the sensor current drops from 4 mA to a level below the reference level of 2 mA, the op-amp will respond with a “low” output voltage that is typically about 0.1 V. The output of the operational amplifier will then signal the alarm circuit about the disturbance. Note from the above that it is not necessary for the sensor current to drop to 0 mA to signal the alarm circuit—just a variation around the reference level that appears unusual.

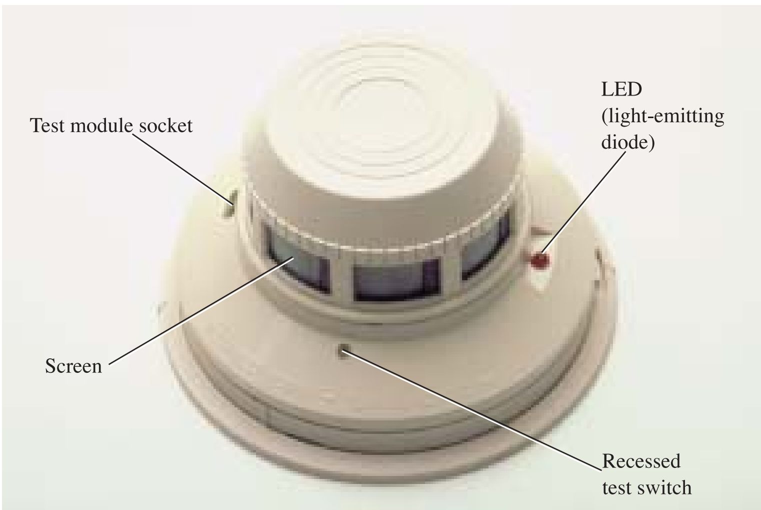
One very important characteristic of this particular op-amp is that the input impedance to the op-amp is relatively low. This feature is important because you don’t want alarm circuits reacting to every voltage spike or turbulence that comes down the line because of external switching action or outside forces such as lightning. In Fig. 8.88(c), for instance, if a high voltage should appear at the input to the series configuration, most of the voltage will be absorbed by the series resistance of the sensor circuit rather than traveling across the input terminals of the operational amplifier—thus preventing a false output and an activation of the alarm.

Wheatstone Bridge Smoke Detector

The Wheatstone bridge is a popular network configuration whenever detection of small changes in a quantity is required. In Fig. 8.89(a), the dc bridge configuration is employing a photoelectric device to detect the presence of smoke and to sound the alarm. A photograph of an actual photoelectric smoke detector appears in Fig. 8.89(b), and the internal construction of the unit is shown in Fig. 8.89(c). First note that air vents are provided to permit the smoke to enter the chamber below the clear plastic. The clear plastic will prevent the smoke from entering the upper chamber but will permit the light

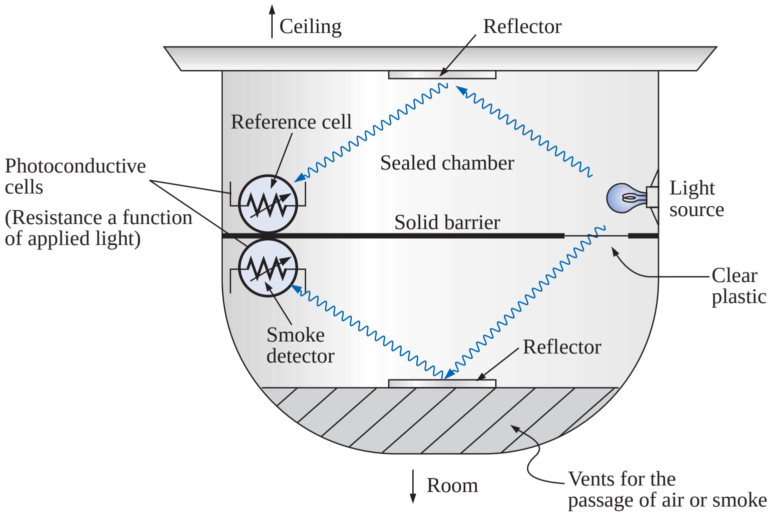


(a)



(b)

FIG. 8.89(a)(b)
Wheatstone bridge detector: (a) dc bridge configuration; (b) outside appearance.



(c)

FIG. 8.89(c)
Wheatstone bridge smoke detector: (c) internal construction.

from the bulb in the upper chamber to bounce off the lower reflector to the semiconductor light sensor (a cadmium photocell) at the left side of the chamber. The clear plastic separation ensures that the light hitting the light sensor in the upper chamber is not affected by the entering smoke. It establishes a reference level to compare against the chamber with the entering smoke. If no smoke is present, the difference in response between the sensor cells will be registered as the normal situation. Of course, if both cells were exactly identical, and if the clear plastic did not cut down on the light, both sensors would establish the same reference level, and their difference would be zero. However, this is seldom the case, so a reference difference is recognized as the sign that smoke is not present. However, once smoke is present, there will be a sharp difference in the sensor reaction from the norm, and the alarm should be sounded.

In Fig. 8.89(a), we find that the two sensors are located on opposite arms of the bridge. With no smoke present the balance-adjust rheostat will be used to ensure that the voltage V between points a and b is zero volts and the resulting current through the primary of the sensitive relay will be zero amperes. Taking a look at the relay, we find that the absence of a voltage from a to b will leave the relay coil unenergized and the switch in the N/O position (recall that the position of a relay switch is always drawn in the unenergized state). An unbalanced situation will result in a voltage across the coil and activation of the relay, and the switch will move to the N/C position to complete the alarm circuit and activate the alarm. Relays with two contacts and one movable arm are called *single-pole–double-throw* (SPDT) relays. The dc power is required to set up the balance situation, energize the parallel bulb so we know that the system is on, and provide the voltage from a to b if an unbalanced situation should develop.

One may ask why only one sensor isn't used since its resistance would be sensitive to the presence of smoke. The answer lies in the fact that the smoke detector might generate a false readout if the supply voltage or output light intensity of the bulb should vary. Smoke detectors of the type just described must be used in gas stations, kitchens, dentist offices, etc., where the range of gas fumes present may set off an ionizing type smoke detector.

Schematic with Nodal Voltages

When an investigator is presented with a system that is down or not operating properly, one of the first options is to check the system's specified voltages on the schematic. These specified voltage levels are actually the nodal voltages determined in this chapter. *Nodal voltage* is simply a special term for a voltage measured from that point to ground. The technician will attach the negative or lower-potential lead to the ground of the network (often the chassis) and then place the positive or higher-potential lead on the specified points of the network to check the nodal voltages. If they match, it is a good sign that that section of the system is operating properly. If one or more fail to match the given values, the problem area can usually be identified. Be aware that a reading of -15.87 V is significantly different from an expected reading of $+16$ V if the leads have been properly attached. Although the actual numbers seem close, the difference is actually more than 30 V. One must expect some deviation from the given value as shown, but always be very sensitive to the resulting sign of the reading.

The schematic of Fig. 8.90(a) includes the nodal voltages for a logic probe used to measure the input and output states of integrated circuit logic chips. In other words, the probe determines whether the measured voltage is one of two states: high or low (often referred to as "on" or "off" or 1 or 0). If the LOGIC IN terminal of the probe is placed on a chip at a location where the voltage is between 0 and 1.2 V, the voltage is considered a low level, and the green LED will light. (LEDs are light-emitting semiconductor diodes that will emit light when current is passed through them.) If the measured voltage is between 1.8 V and 5 V, the reading is considered high, and the red LED will light. Any voltage between 1.2 V and 1.8 V is considered a "floating level" and is an indication that the system being measured is not operating correctly. Note that the reference levels mentioned above are established by the voltage divider network to the right of the schematic. The op-amps employed are of such high input impedance that their loading on the voltage divider network can be ignored and the voltage divider network considered a network unto itself. Even though three 5.5-V dc supply voltages are indicated on the diagram, be aware that all three points are connected to the same supply. The other voltages provided (the nodal voltages) are the voltage levels that should be present from that point to ground if the system is working properly.

The op-amps are used to sense the difference between the reference at points 3 and 6 and the voltage picked up in LOGIC IN. Any difference will result in an output that will light either the green or the red LED. Be aware, because of the direct connection, that the voltage at point 3 is the same as shown by the nodal voltage to the left, or 1.8 V. Likewise, the voltage at point 6 is 1.2 V for comparison with the voltages at points 5 and 2, which reflect the measured voltage. If the input voltage happened to be 1.0 V, the difference between the voltages at points 5 and 6 would be 0.2 V, which ideally would appear at point 7. This low potential at point 7 would result in a current flowing from the much higher 5.5-V dc supply through the green LED, causing it to light and indicating a low condition. By the way, LEDs, like diodes, permit current through them only in the direction of the arrow in the symbol. Also note that the voltage at point 6 must be higher than that at point 5 for the output to turn on the LED. The same is true for point 2 over point 3, which reveals why the red LED does not light when the 1.0-V level is measured.

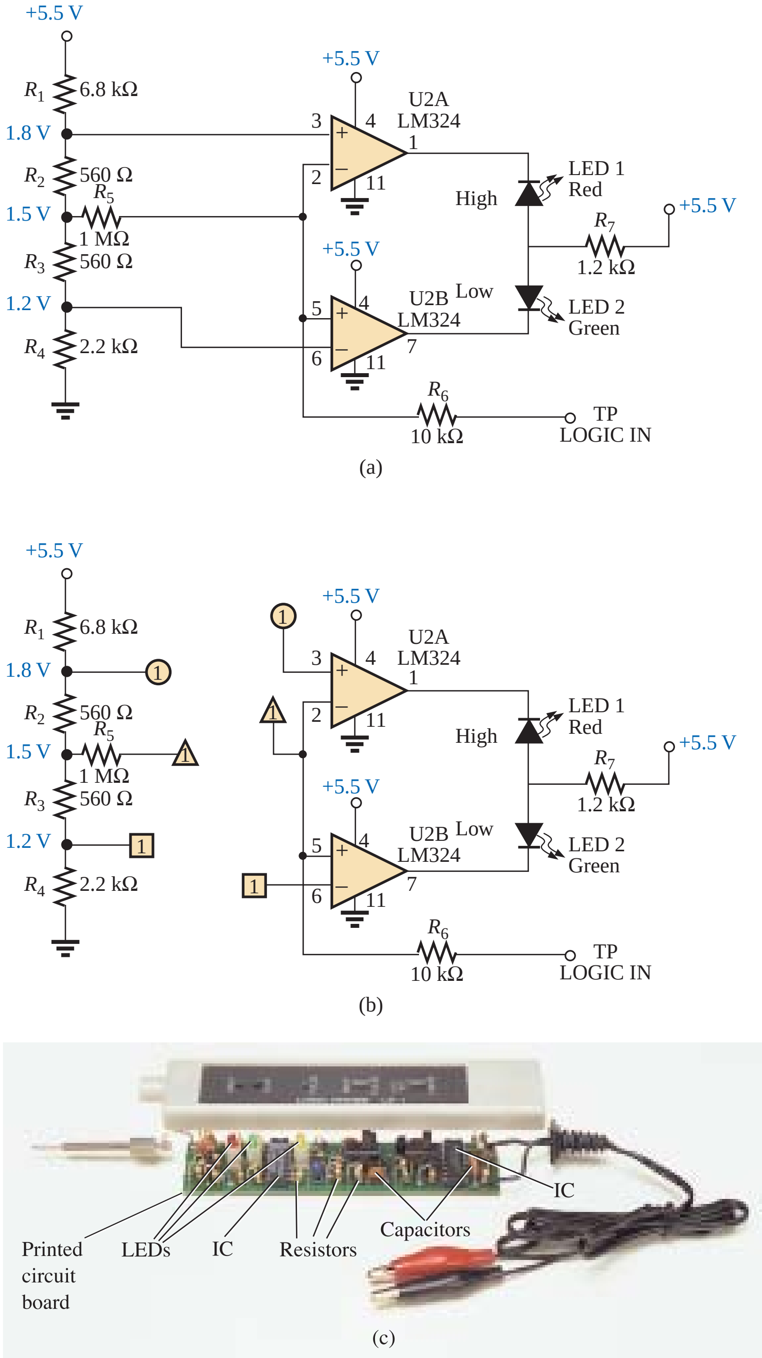


FIG. 8.90

Logic probe: (a) schematic with nodal voltages; (b) network with global connections; (c) photograph of commercially available unit.

Oftentimes it is impractical to draw the full network as shown in Fig. 8.90(b) because there are space limitations or because the same voltage divider network is used to supply other parts of the system. In such cases one must recognize that points having the same shape are connected, and the number in the figure reveals how many connections are made to that point.

A photograph of the outside and inside of a commercially available logic probe is provided in Fig. 8.90(c). Note the increased complexity of system because of the variety of functions that the probe can perform.

8.14 COMPUTER ANALYSIS

PSpice

The bridge network of Fig. 8.70 will now be analyzed using PSpice to ensure that it is in the balanced state. The only component that has not been introduced in earlier chapters is the dc current source. It can be obtained by first selecting the **Place a part** key and then the **SOURCE** library. Scrolling the **Part List** will result in the option **IDC.** A left click of **IDC** followed by **OK** will result in a dc current source whose direction is toward the bottom of the screen. One left click of the mouse (to make it red—active) followed by a right click of the mouse will result in a listing having a **Mirror Vertically** option. Selecting that option will flip the source and give it the direction of Fig. 8.70.

The remaining parts of the PSpice analysis are pretty straightforward, with the results of Fig. 8.91 matching those obtained in the analysis of Fig. 8.70. The voltage across the current source is 8 V positive to ground, and the voltage at either end of the bridge arm is 2.667 V. The voltage across R_5 is obviously 0 V for the level of accuracy displayed, and the current is of such a small magnitude compared to the other current levels of the network that it can essentially be considered 0 A. Note also for the balanced bridge that the current through R_1 equals that of R_3, and the current through R_2 equals that of R_4.

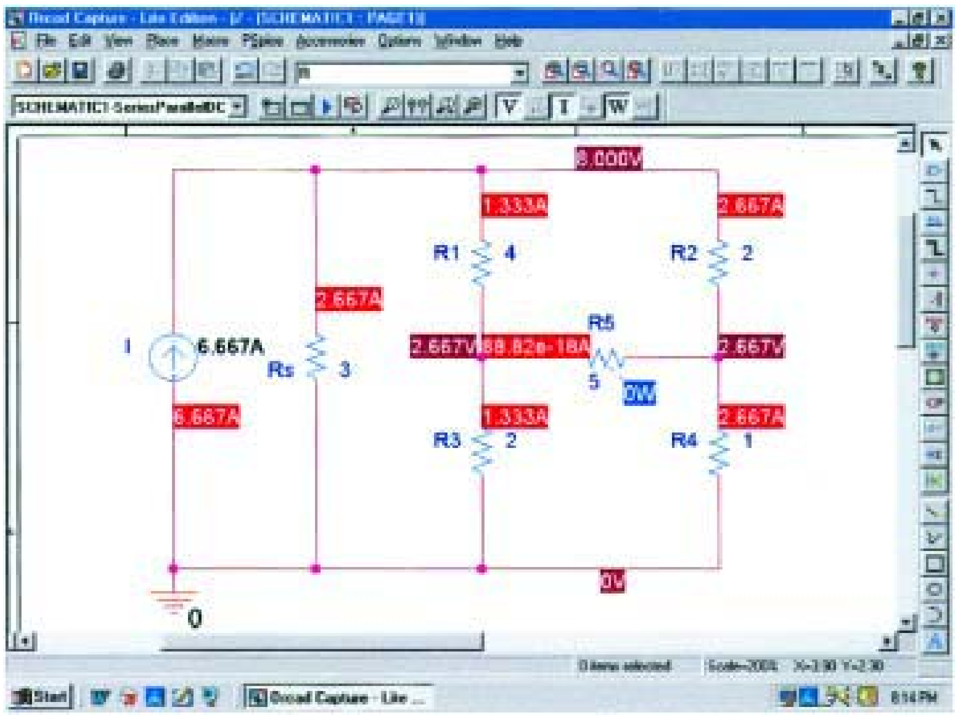


FIG. 8.91
Applying PSpice to the bridge network of Fig. 8.70.

Electronics Workbench

Electronics Workbench will now be used to verify the results of Example 8.18. All the elements of creating the schematic of Fig. 8.92 have been presented in earlier chapters; they will not be repeated here in order to demonstrate how little documentation is now necessary to carry you through a fairly complex network.

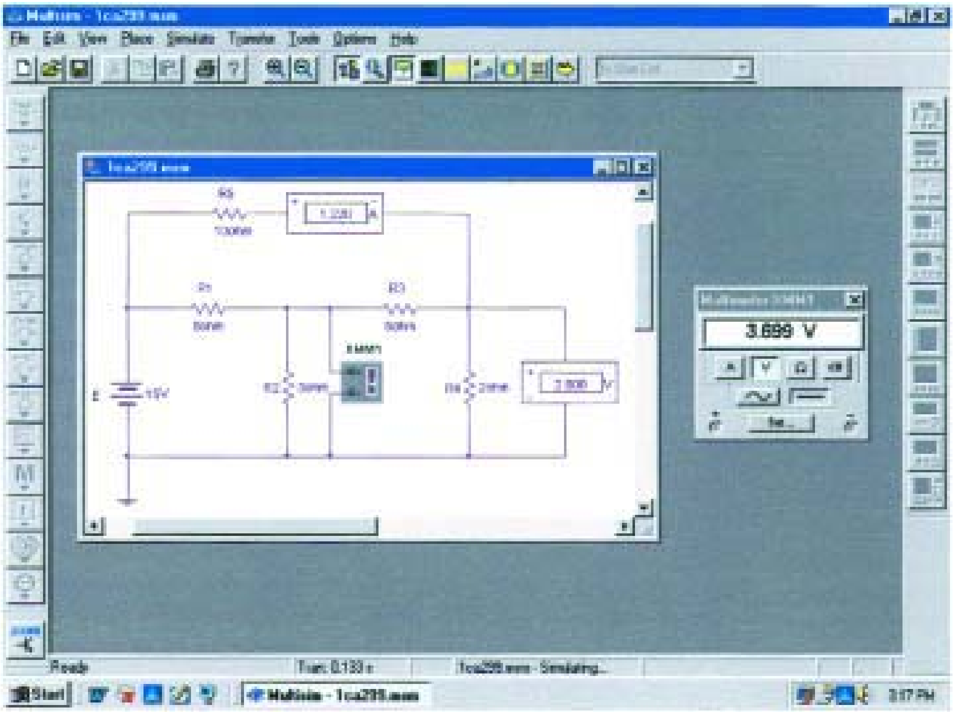


FIG. 8.92
Using Electronics Workbench to verify the results of Example 8.18.

For the analysis, both indicators and a meter will be used to display the desired results. An **A** indicator in the **H** position was used for the current through R_5, and a **V** indicator in the **V** position was used for the voltage across R_2. A multimeter in the voltmeter mode was placed to read the voltage across R_4. The ammeter is reading the mesh or loop current for that branch, and the two voltmeters are displaying the nodal voltages of the network.

After simulation, the results displayed are an exact match with those of Example 8.18.

PROBLEMS

SECTION 8.2 Current Sources

1. Find the voltage V_{ab} (with polarity) across the ideal current source of Fig. 8.93.

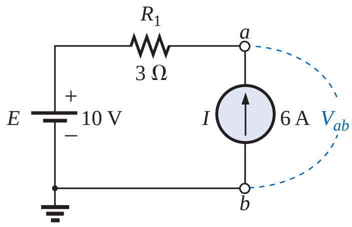


FIG. 8.93
Problem 1.

2. **a.** Determine V for the current source of Fig. 8.94(a) with an internal resistance of 10 kΩ.
   b. The source of part (a) is approximated by an ideal current source in Fig. 8.94(b) since the source resistance is much larger than the applied load. Determine the resulting voltage V for Fig. 8.94(b), and compare it to that obtained in part (a). Is the use of the ideal current source a good approximation?

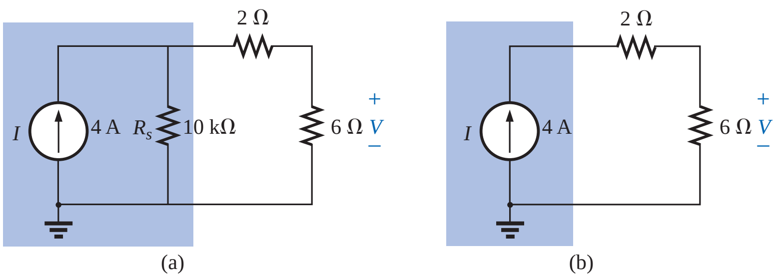


FIG. 8.94
Problem 2.

3. For the network of Fig. 8.95:
   a. Find the currents I_1 and I_s.
   b. Find the voltages V_s and V_3.

4. Find the voltage V_3 and the current I_2 for the network of Fig. 8.96.

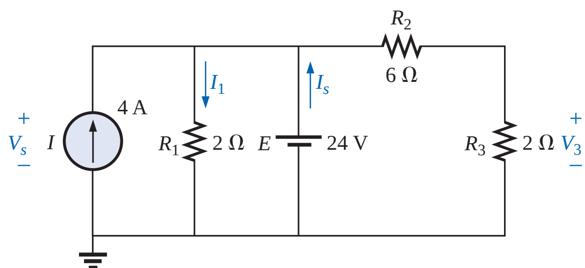


FIG. 8.95
Problem 3.

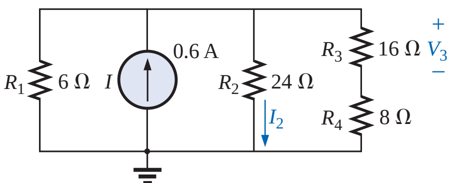


FIG. 8.96
Problem 4.

SECTION 8.3 Source Conversions

5. Convert the voltage sources of Fig. 8.97 to current sources.

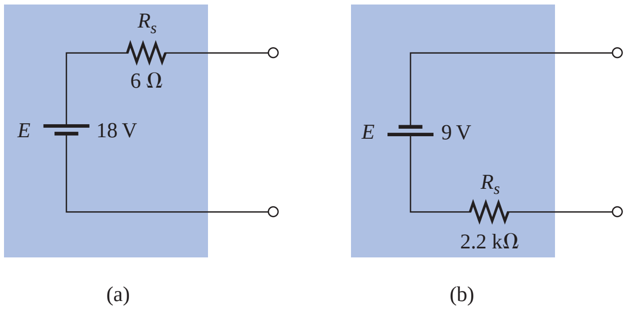


FIG. 8.97
Problem 5.

6. Convert the current sources of Fig. 8.98 to voltage sources.

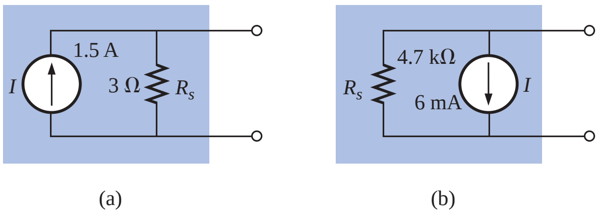


FIG. 8.98
Problem 6.

7. For the network of Fig. 8.99:

a. Find the current through the 2-Ω resistor.

b. Convert the current source and 4-Ω resistor to a voltage source, and again solve for the current in the 2-Ω resistor. Compare the results.

8. For the configuration of Fig. 8.100:

a. Convert the current source and 6.8-Ω resistor to a voltage source.

b. Find the magnitude and direction of the current I_1.

c. Find the voltage V_{ab} and the polarity of points a and b.

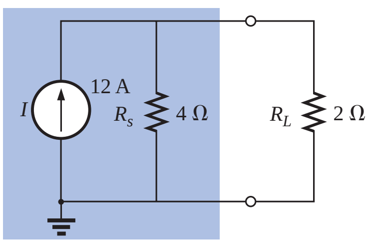


FIG. 8.99
Problem 7.

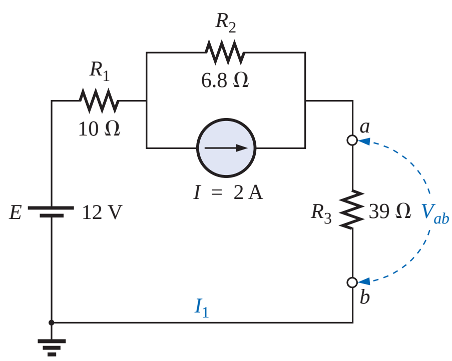


FIG. 8.100
Problem 8.

SECTION 8.4 Current Sources in Parallel

9. Find the voltage V_2 and the current I_1 for the network of Fig. 8.101.

10. **a.** Convert the voltage sources of Fig. 8.102 to current sources.
b. Find the voltage V_{ab} and the polarity of points a and b.
c. Find the magnitude and direction of the current I.

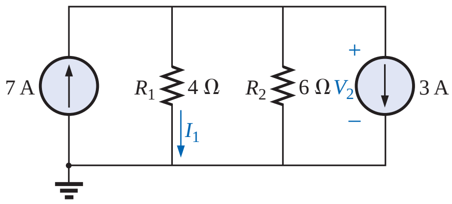


FIG. 8.101
Problem 9.

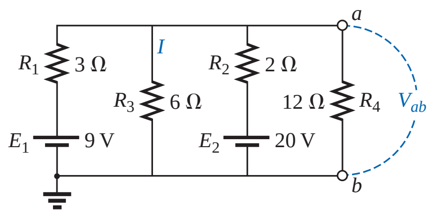


FIG. 8.102
Problem 10.

11. For the network of Fig. 8.103:
a. Convert the voltage source to a current source.
b. Reduce the network to a single current source, and determine the voltage V_1.
c. Using the results of part (b), determine V_2.
d. Calculate the current I_2.

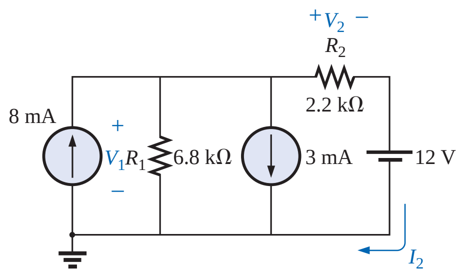


FIG. 8.103
Problem 11.

SECTION 8.6 Branch-Current Analysis

12. Using branch-current analysis, find the magnitude and direction of the current through each resistor for the networks of Fig. 8.104.

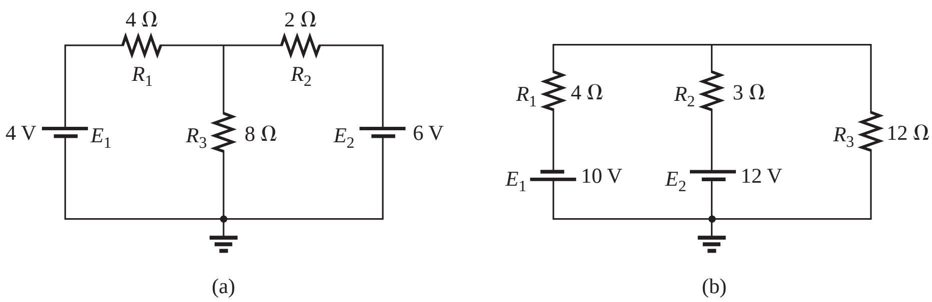


FIG. 8.104
Problems 12, 17, 25, and 54.

***13.** Using branch-current analysis, find the current through each resistor for the networks of Fig. 8.105. The resistors are all standard values.

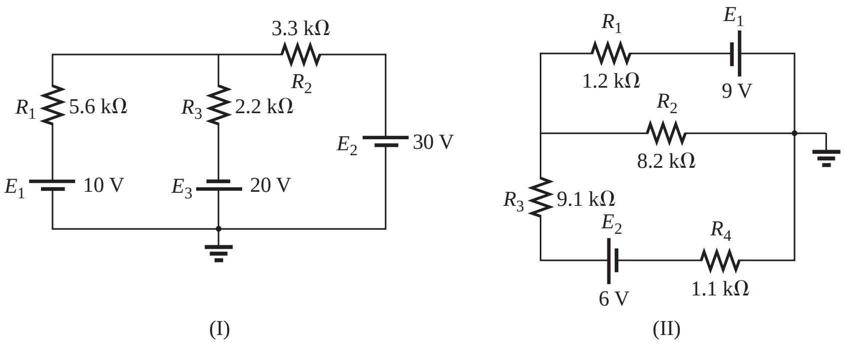


FIG. 8.105
Problems 13, 18, and 26.

***14.** For the networks of Fig. 8.106, determine the current I_2 using branch-current analysis, and then find the voltage V_{ab}.

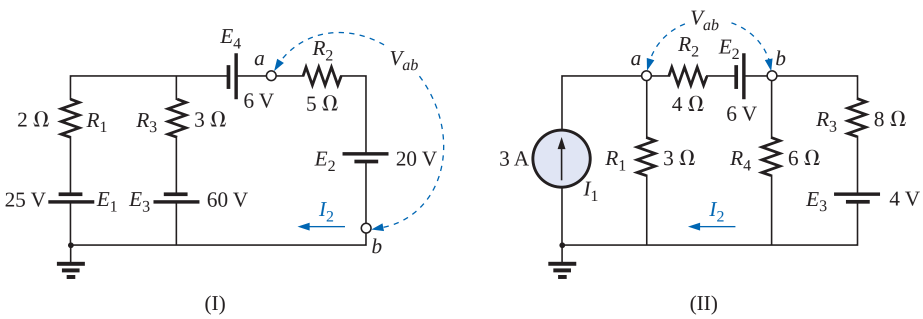


FIG. 8.106
Problems 14, 19, and 27.

***15.** For the network of Fig. 8.107:

a. Write the equations necessary to solve for the branch currents.

b. By substitution of Kirchhoff's current law, reduce the set to three equations.

c. Rewrite the equations in a format that can be solved using third-order determinants.

d. Solve for the branch current through the resistor R_3.

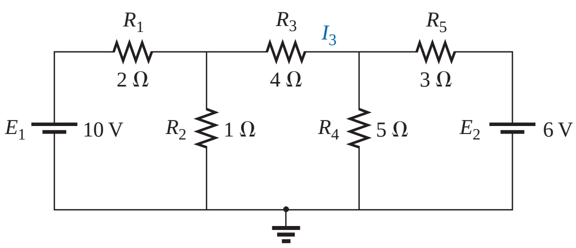


FIG. 8.107
Problems 15, 20, and 28.

***16.** For the transistor configuration of Fig. 8.108:

a. Solve for the currents I_B, I_C, and I_E using the fact that $V_{BE} = 0.7$ V and $V_{CE} = 8$ V.

b. Find the voltages V_B, V_C, and V_E with respect to ground.

c. What is the ratio of output current I_C to input current I_B? [*Note:* In transistor analysis this ratio is referred to as the *dc beta* of the transistor (β_{dc}).]

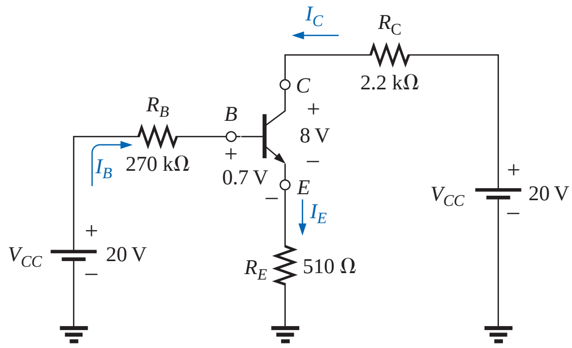


FIG. 8.108
Problem 16.

SECTION 8.7 Mesh Analysis (General Approach)

17. Find the current through each resistor for the networks of Fig. 8.104.

18. Find the current through each resistor for the networks of Fig. 8.105.

19. Find the mesh currents and the voltage V_{ab} for each network of Fig. 8.106. Use clockwise mesh currents.

20. a. Find the current I_3 for the network of Fig. 8.107 using mesh analysis.

b. Based on the results of part (a), how would you compare the application of mesh analysis to the branch-current method?

***21.** Using mesh analysis, determine the current through the 5-Ω resistor for each network of Fig. 8.109. Then determine the voltage V_a.

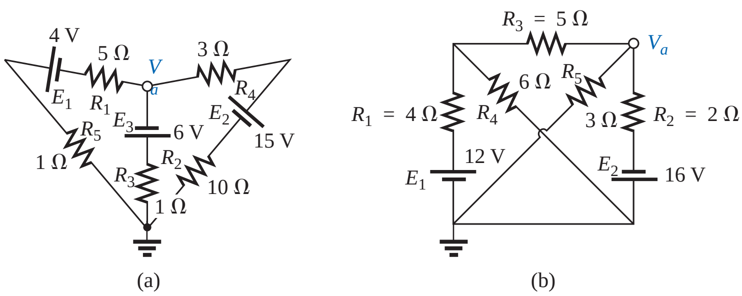


FIG. 8.109
Problems 21 and 29.

***22.** Write the mesh equations for each of the networks of Fig. 8.110, and, using determinants, solve for the loop currents in each network. Use clockwise mesh currents.

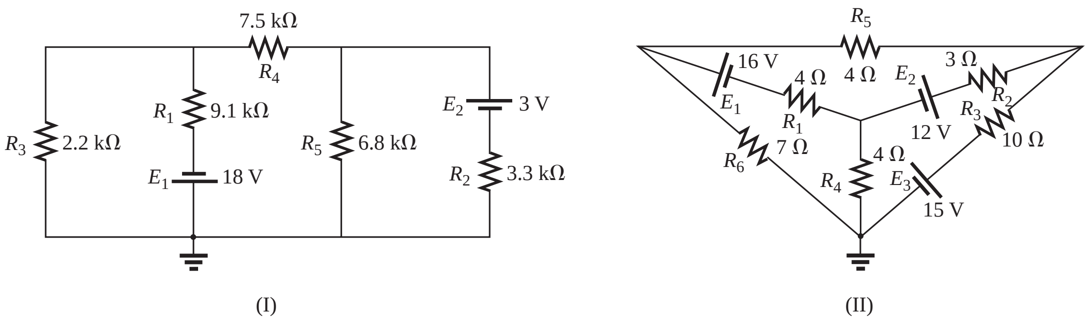


FIG. 8.110
Problems 22, 30, and 34.

***23.** Write the mesh equations for each of the networks of Fig. 8.111, and, using determinants, solve for the loop currents in each network.

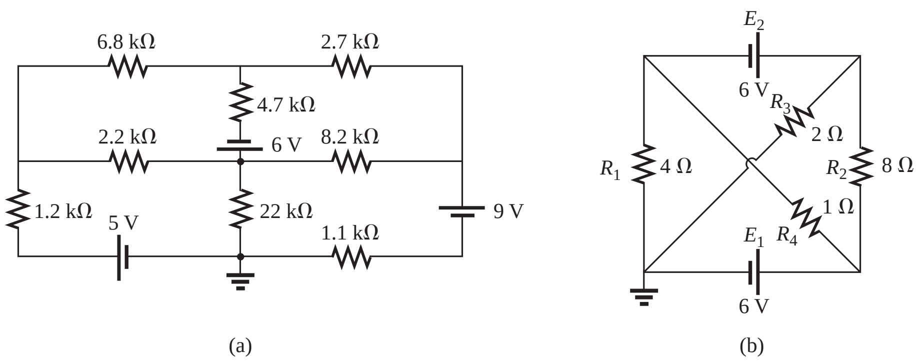


FIG. 8.111
Problems 23, 31, and 55.

***24.** Using the supermesh approach, find the current through each element of the networks of Fig. 8.112.

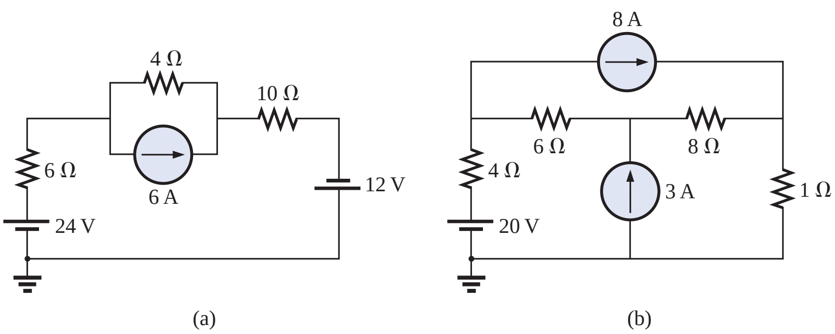


FIG. 8.112
Problem 24.

SECTION 8.8 Mesh Analysis (Format Approach)

25. Using the format approach, write the mesh equations for the networks of Fig. 8.104. Is symmetry present? Using determinants, solve for the mesh currents.

26. **a.** Using the format approach, write the mesh equations for the networks of Fig. 8.105.
b. Using determinants, solve for the mesh currents.
c. Determine the magnitude and direction of the current through each resistor.

27. **a.** Using the format approach, write the mesh equations for the networks of Fig. 8.106.
b. Using determinants, solve for the mesh currents.
c. Determine the magnitude and direction of the current through each resistor.

28. Using mesh analysis, determine the current I_3 for the network of Fig. 8.107, and compare your answer to the solution of Problem 15.

29. Using mesh analysis, determine $I_{5\Omega}$ and V_a for the network of Fig. 8.109(b).

30. Using mesh analysis, determine the mesh currents for the networks of Fig. 8.110.

31. Using mesh analysis, determine the mesh currents for the networks of Fig. 8.111.

SECTION 8.9 Nodal Analysis (General Approach)

32. Write the nodal equations for the networks of Fig. 8.113, and, using determinants, solve for the nodal voltages. Is symmetry present?

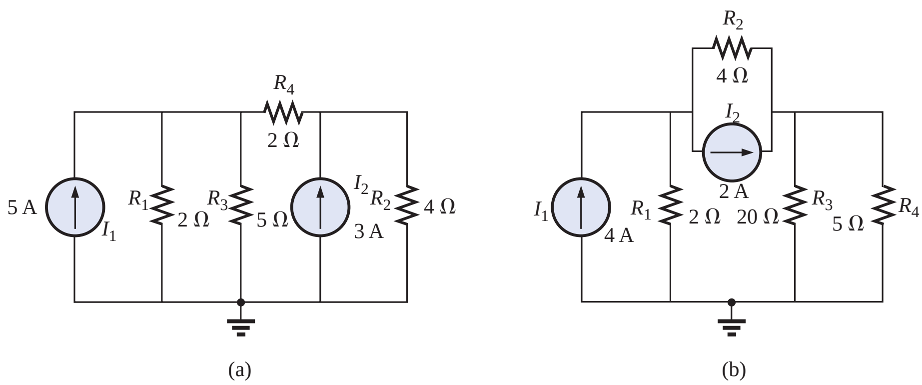


FIG. 8.113
Problems 32 and 38.

33. a. Write the nodal equations for the networks of Fig. 8.114.
b. Using determinants, solve for the nodal voltages.
c. Determine the magnitude and polarity of the voltage across each resistor.

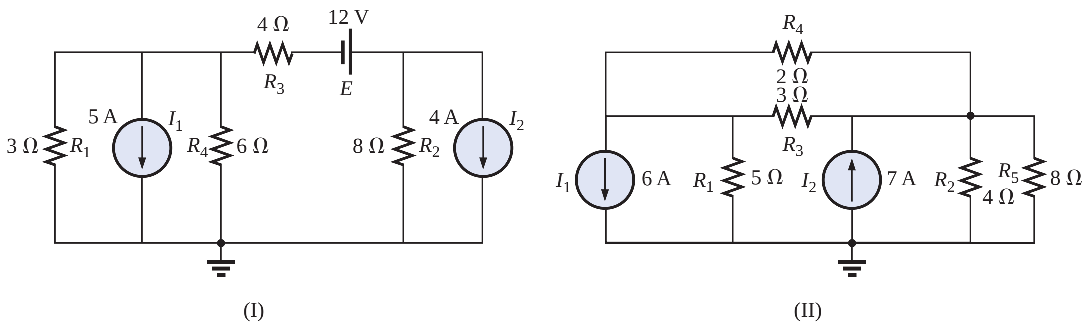


FIG. 8.114
Problems 33 and 39.

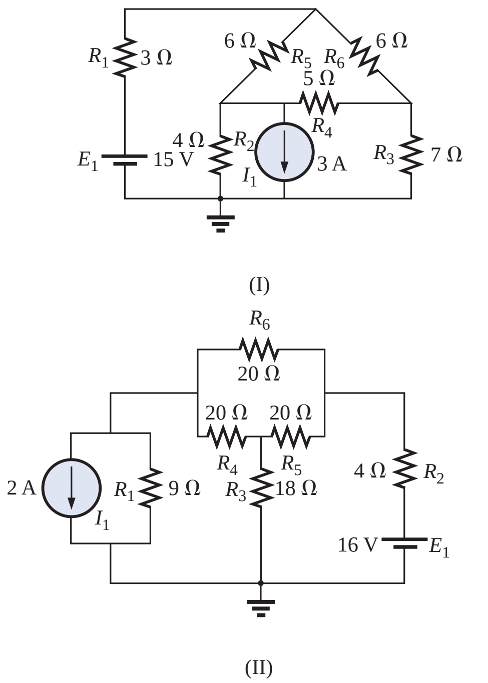


FIG. 8.115
Problems 35 and 40.

34. a. Write the nodal equations for the networks of Fig. 8.110.
b. Using determinants, solve for the nodal voltages.
c. Determine the magnitude and polarity of the voltage across each resistor.

***35.** For the networks of Fig. 8.115, write the nodal equations and solve for the nodal voltages.

36. a. Determine the nodal voltages for the networks of Fig. 8.116.
b. Find the voltage across each current source.

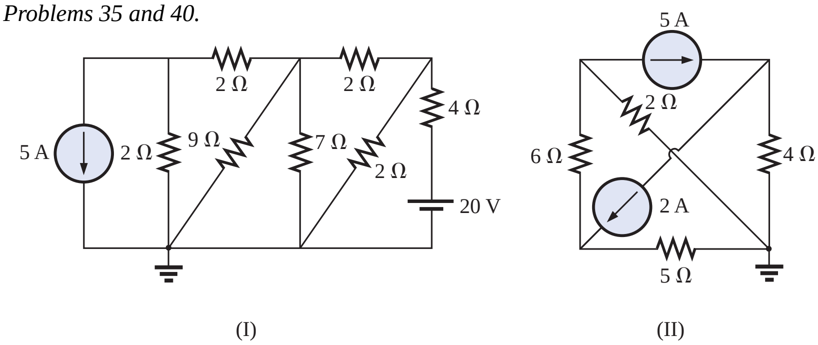


FIG. 8.116
Problems 36 and 41.

***37.** Using the supernode approach, determine the nodal voltages for the networks of Fig. 8.117.

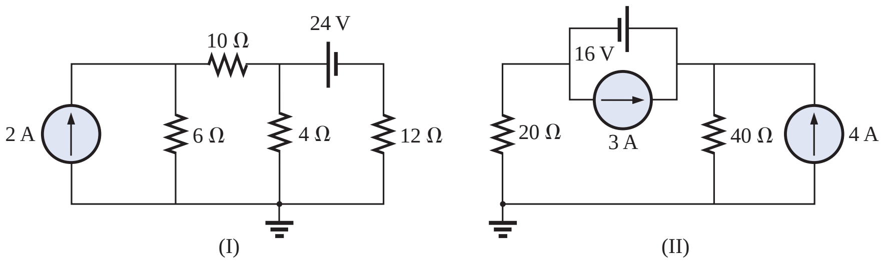


FIG. 8.117
Problems 37 and 56.

SECTION 8.10 Nodal Analysis (Format Approach)

38. Using the format approach, write the nodal equations for the networks of Fig. 8.113. Is symmetry present? Using determinants, solve for the nodal voltages.

39. a. Write the nodal equations for the networks of Fig. 8.114.
b. Solve for the nodal voltages.
c. Find the magnitude and polarity of the voltage across each resistor.

40. a. Write the nodal equations for the networks of Fig. 8.115.
b. Solve for the nodal voltages.
c. Find the magnitude and polarity of the voltage across each resistor.

41. Determine the nodal voltages for the networks of Fig. 8.116. Then determine the voltage across each current source.

SECTION 8.11 Bridge Networks

42. For the bridge network of Fig. 8.118:
a. Write the mesh equations using the format approach.
b. Determine the current through R_5.
c. Is the bridge balanced?
d. Is Equation (8.4) satisfied?

43. For the network of Fig. 8.118:
a. Write the nodal equations using the format approach.
b. Determine the voltage across R_5.
c. Is the bridge balanced?
d. Is Equation (8.4) satisfied?

44. For the bridge of Fig. 8.119:
a. Write the mesh equations using the format approach.
b. Determine the current through R_5.
c. Is the bridge balanced?
d. Is Equation (8.4) satisfied?

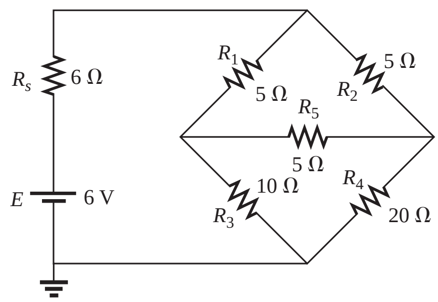


FIG. 8.118
Problems 42 and 43.

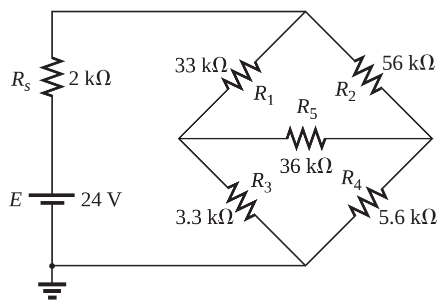


FIG. 8.119
Problems 44 and 45.

45. For the bridge network of Fig. 8.119:
  a. Write the nodal equations using the format approach.
  b. Determine the current across R_5.
  c. Is the bridge balanced?
  d. Is Equation (8.4) satisfied?

46. Write the nodal equations for the bridge configuration of Fig. 8.120. Use the format approach.

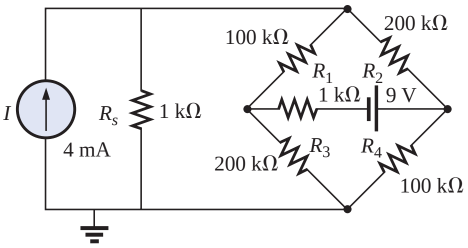


FIG. 8.120
Problem 46.

***47.** Determine the current through the source resistor R_s of each network of Fig. 8.121 using either mesh or nodal analysis. Discuss why you chose one method over the other.

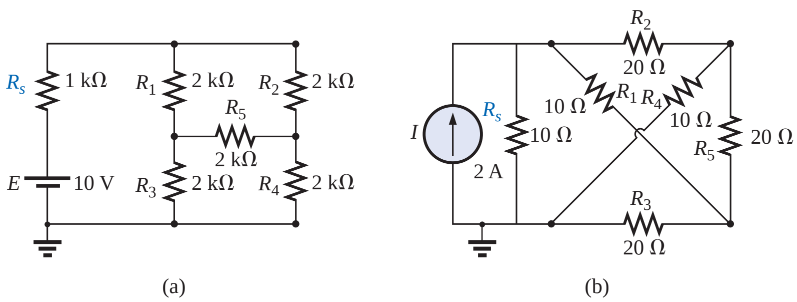


FIG. 8.121
Problem 47.

SECTION 8.12 Y-Δ (T-π) and Δ-Y (π-T) Conversions

48. Using a Δ-Y or Y-Δ conversion, find the current I in each of the networks of Fig. 8.122.

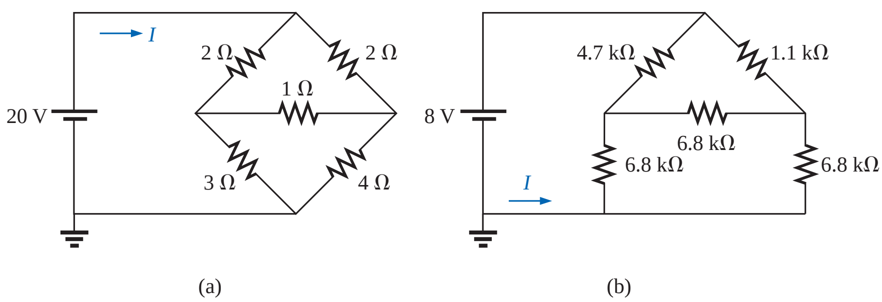


FIG. 8.122
Problem 48.

***49.** Repeat Problem 48 for the networks of Fig. 8.123.

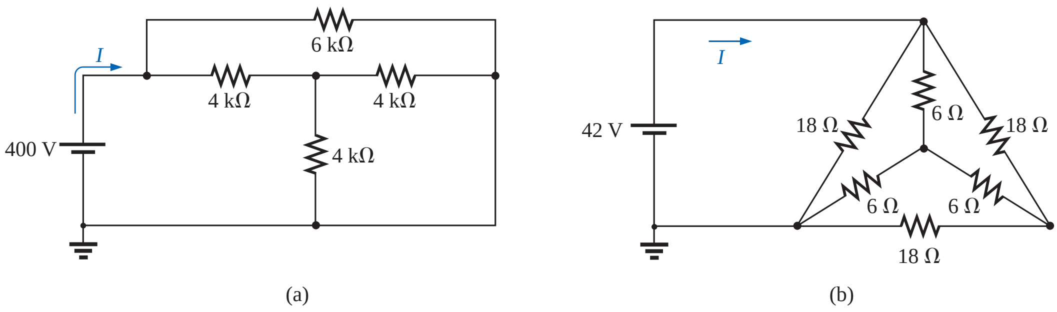


FIG. 8.123
Problem 49.

***50.** Determine the current I for the network of Fig. 8.124.

***51. a.** Replace the T configuration of Fig. 8.125 (composed of 6-kΩ resistors) with a π configuration.

b. Solve for the source current I_{s_1}.

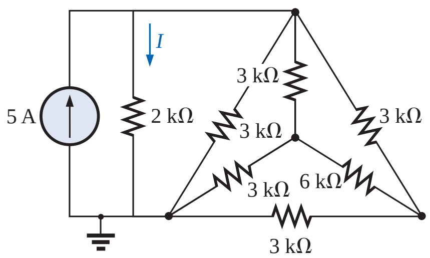


FIG. 8.124
Problem 50.

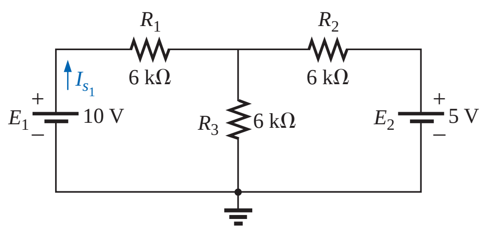


FIG. 8.125
Problem 51.

***52. a.** Replace the π configuration of Fig. 8.126 (composed of 3-kΩ resistors) with a T configuration.

b. Solve for the source current I_s.

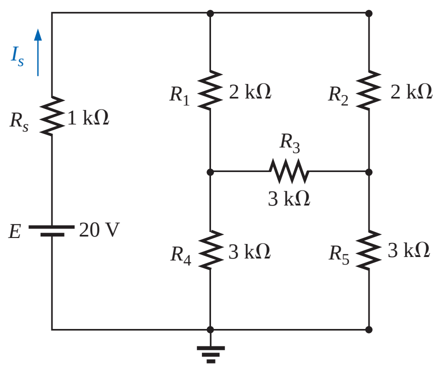


FIG. 8.126
Problem 52.

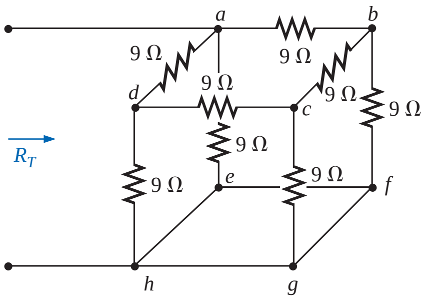


FIG. 8.127
Problem 53.

***53.** Using Y-Δ or Δ-Y conversions, determine the total resistance of the network of Fig. 8.127.

SECTION 8.14 Computer Analysis

PSpice or Electronics Workbench

54. Using schematics, find the current through each element of Fig. 8.104.

***55.** Using schematics, find the mesh currents for the network of Fig. 8.111(a).

***56.** Using schematics, determine the nodal voltages for the network of Fig. 8.117(II).

Programming Language (C++, QBASIC, Pascal, etc.)

57. Given two simultaneous equations, write a program to solve for the unknown variables.

***58.** Using mesh analysis and determinants, write a program to solve for both mesh currents of the network of Fig. 8.26 (for any component values).

***59.** Using nodal analysis and determinants, write a program to solve for the nodal voltages of the network of Fig. 8.44 (for any component values).

GLOSSARY

Branch-current method A technique for determining the branch currents of a multiloop network.

Bridge network A network configuration typically having a diamond appearance in which no two elements are in series or parallel.

Current sources Sources that supply a fixed current to a network and have a terminal voltage dependent on the network to which they are applied.

Delta (Δ), pi (π) configuration A network structure that consists of three branches and has the appearance of the Greek letter delta (Δ) or pi (π).

Determinants method A mathematical technique for finding the unknown variables of two or more simultaneous linear equations.

Mesh analysis A technique for determining the mesh (loop) currents of a network that results in a reduced set of equations compared to the branch-current method.

Mesh (loop) current A labeled current assigned to each distinct closed loop of a network that can, individually or in combination with other mesh currents, define all of the branch currents of a network.

Nodal analysis A technique for determining the nodal voltages of a network.

Node A junction of two or more branches in a network.

Wye (Y), tee (T) configuration A network structure that consists of three branches and has the appearance of the capital letter Y or T.